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A
SHORT ARITHMETIC

PART FIRST:

THE FOUR RULES

ESPECIALLY PREPARED FOR BEGINNERS AT SCHOOL AND
FOR THOSE DESIROUS OF GIVING INSTRUCTION
AT HOME

BY
EDWARD ROTH A.M.

PHILADELPHIA
No. 1135 PINE STREET
1884

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The present work prefers developing the capacity of the ordinary PUPIL by constantly appealing to his judgment, rather than to humoring his indolence by making things too easy. It clears away all difficulties, but always tries to keep his watchfulness on the alert. It attempts to teach him great principles by first enabling him to find them out for himself, and then by keeping him at their application as long as the Teacher deems necessary.

It helps the TEACHER, too. Regularly making the Explanation a most important and unavoidable point in each lesson, it spares him endless questions and troublesome repetitions. The Pupil, forewarned as to his probable mistakes, will try to avoid them; and when he makes them, instead of waiting helplessly for aid, he will set about correcting them intelligently himself. Being also obliged to *prove* his work, whenever ready proof is possible, he will know, himself, when and where he is wrong, without waiting to be told so by the Teacher.

Finally, the work is particularly intended for the help of every PARENT or other HOME INSTRUCTOR who wishes to give the children intelligent aid in an important study, which he values very highly, but much of which he must naturally have forgotten.

To carry out the idea constantly aimed at in this work, the Teacher, *first*, makes the Pupils read the lesson aloud in class, going over it several times, if necessary. *Then*, giving whatever other explanation he deems essential, and dwelling on it until it is well understood, the Teacher, taking the Model for an illustration, questions each pupil separately on the subject of the lesson, demanding a ready and intelligent answer. *Lastly*, it is only when he is satisfied that the majority of the class has mastered every difficulty, but not till then, that he allows the Pupils to attempt the Practical Questions.

When the Pupil enters the Practical Questions in his copybook, he should number each one carefully, and strictly imitate the Model in every respect. *This should be insisted on.* Carelessness begets carelessness. The careless arithmetician is like the careless talker—his evidence is untrustworthy: no one knows when he is telling the truth.

PHILADELPHIA, *April* 15, 1884.

☞ The addition of PROBLEMS, ANALYSIS, BILLS, and EXAMINATION QUESTIONS renders PART FIRST quite a little treatise as far as it goes. Of course, it is intended for such pupils only as understand already how to read, write, and make figures with tolerable correctness.

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A SHORT ARITHMETIC.

PART I.

THE FOUR RULES.

CHAPTER I.

THE METHODS. NUMERATION. NOTATION.

1. *Why do you learn Arithmetic?*

To understand the science of Numbers so as to be able to calculate by them.

2. *What is a Number?*

A Number is a unit (one) or a collection of units. For example: *Three* is a number, being three ones; and *Ten* is a number, being ten ones. Numbers are usually regarded as *Concrete* or *Abstract.*

3. *What is a Concrete Number?*

A Number is called Concrete when it is united with some object; example: *six boys, three desks, four apples;* here the Numbers are Concrete, because they denote a certain number of objects. A Denominate Number means the same thing.

4. *What is an Abstract Number?*

A Number is called Abstract when it is not united with any particular object; example: *six, three, four, ten,* etc., are Abstract Numbers.

5. What is the first thing to be learned in Arithmetic?

As in Reading we first learn to understand and write the Letters, so in Arithmetic we first learn to understand and write the Numbers. In order to be able to name and understand numbers we learn *Numeration;* and in order to be able to write them we learn *Notation.*

6. How are numbers written?

Chiefly in two ways. The modern way to write numbers is by using certain marks called *figures;* this is called the Arabic method. The other way is by using certain *letters* of the alphabet to denote certain numbers; this is called the Roman method. All calculations are now made by the Arabic method, but the Roman is still employed often enough to oblige us to understand its principles.

It is called Roman because it was the mode of calculation employed by the ancient Romans.

7. What was the leading principle of the Roman method?

The main principle of the Roman method was to express numbers by *Addition* or *Subtraction.* To express the different numbers, the Romans employed only seven letters, all capitals; namely, I, V, X, L, C, D, and M. I represented one; V, five; X, ten; L, fifty; C, one hundred; D, five hundred; and M, one thousand.

8. How were the other Numbers expressed?

The other numbers were expressed by certain combinations of these letters, according to fixed rules.

FIRST RULE.—*Numbers placed at the right of Numbers greater than themselves were to be added to them.* Examples: VI was six; XII = twelve; III = three; XX = twenty; LV = fifty-five; and CX = one hundred and ten.

SECOND RULE.—*Numbers placed at the left of Numbers greater than themselves were to be subtracted from them.* Examples: IV = four; IX = nine; XL = forty; XC = ninety.

9. *Were these rules always strictly carried out?*

Not always. V, for example, was never placed on the left of any letter; *forty-five* was not represented by VL, but by XLV. Likewise I was seldom placed at the left of any greater number than V or X; example: *forty-nine* was represented, not by IL, but by XLIX.

10. *How did the Romans avoid a great accumulation of characters in the higher numbers?*

To prevent too great an accumulation of characters in the higher numbers, a bar (—) turned into so many thousands the number over which it was placed; example: $\overline{\text{V}}$ represented *five thousand;* $\overline{\text{X}}$, *ten thousand;* $\overline{\text{LI}}$, *fifty-one thousand;* $\overline{\text{C}}$ *one hundred thousand,* etc.

PRACTICAL QUESTIONS. No. 1.

ROMAN NOTATION AND NUMERATION.

1. Read out what is denoted by the following Roman numerals: VI, IX, XXIX, XLVI, LXXII, LXXXIX, XCV, CCCLXIII, CMLXXXVII, MDCCCLXXXVI, $\overline{\text{VII}}$, $\overline{\text{XVIII}}$, $\overline{\text{CV}}$, $\overline{\text{CCVI}}$.

2. Express in Roman characters the following written numbers: *Ninety-eight, twenty-six, fifteen, nine, sixty-three, forty, ninety-two, thirty-four, seventy-seven, nineteen.*

3. Express in Roman characters the following written numbers: *three hundred and twenty-six, eight hundred and sixty-three, one hundred and forty, nine thousand three hundred and one, seventy-six thousand five hundred and forty-seven, seventeen hundred and seventy-six, fourteen hundred and ninety-two, three thousand and twelve.*

Other examples, if necessary, should be given by the Teacher, and, if possible, they should all be done, in presence of the class, on the blackboard. *Nothing should be passed until well known. The effort of the mind necessary to understand a principle is of the greatest importance in learning.*

The Pupil should be made to understand in the beginning that *hurry* is one of the greatest obstacles to real knowledge.

11.* *What is the Arabic method of Notation?

The Arabic differs from the Roman method in two respects: first, instead of *letters*, it uses *signs*, called figures, to represent the numbers; second, the value of the numbers does not depend on the *addition* or *subtraction* of these figures, but strictly on their *position*. It has only ten of these figures, often called *digits*, namely, 0 (nought or zero), 1, 2, 3, 4, 5, 6, 7, 8, and 9; but by using a simple principle in arranging them, even exceedingly high numbers can be expressed with great ease and readiness.

12.* *What is this principle?

This great principle is, *that every figure should be kept strictly in its own place.* The first figure 0, called nought or zero, is in itself of no value, but it is as useful as the others in forcing them to keep their ranks. These ranks begin on the right and are counted *towards the left.* The first rank, called the *unit* rank, can never express more than *nine.* The second rank, called the *ten* rank, can never express more than *ninety.* The third rank, called the *hundred* rank, can never express more than *nine hundred.* Before going any farther, let us take an example. Express CCCLXXXIIII by the Arabic method. We see at once that this number consists of three *hundreds*, eight *tens*, and four *units.* We therefore put 4 in the first rank, 8 in the second, and 3 in the third rank, as follows: 384.

13.* *How is this number read?

It may be read in two ways: one easy on the reader, but long, and therefore hard on the hearer; namely: *four units, eight tens*, and *three hundreds;* the other, more difficult on the reader, but short, and therefore more readily understood by the hearer; namely: *three hundred and eighty-four.*

***14.** At what rate does each rank increase in value?*

Each rank to the left increases the value of its figure tenfold; the Arabic system is, therefore, called *Decimal,* from the Latin word, *decem,* ten, and it is evidently derived from our habit of using the ten fingers in calculation.

***15.** Explain more fully the use of the Nought.*

Suppose I wanted to express *three hundred and four;* evidently 34 would not do, 3 in the second rank meaning 3 *tens* only, not 3 *hundreds.* Therefore I put back 3 to its proper place, the third rank, and keep it there by filling up the second rank with a *Nought,* 304. If I wanted to express *three hundred* only, I would put 3 in the third rank, and keep it there by annexing two *Noughts* to the right, 300.

***16.** Continue your explanation.*

Figures in the fourth rank always represent *thousands;* in the fifth rank, *ten thousands;* in the sixth rank, *hundred thousands;* in the seventh rank, *millions*—every new rank increasing tenfold in the value of its figures over those of the previous one.

It is evident that in any number every figure serves a double purpose: it not only tells its own value, but it also keeps the other figures in their proper places. In 4706, the 7 not only gives us the number of *hundreds,* but, by keeping 4 in its place, it informs us the latter figure represents 4 thousands. The nought here tells us there are no *tens* in the number, and also, by keeping the 7 in the third rank, it compels it to represent 7 *hundreds.*

***17.** What is the name for the ranks after the seventh?*

The eighth contains *ten millions;* the ninth, *hundred millions;* the tenth, *billions;* the eleventh, *ten billions;* and so on as before. The third figure after *billions* denotes *trillions;* the next third, *quadrillions;* the next third, *quintillions,* etc.

***18.** Tell me some short and ready way by which I can always* notate *or* numerate *without much trouble.*

Let us start with *Numeration.* Suppose I am asked to numerate, that is, read off, the following number, consisting of eleven figures: 93428965347. I begin at the right and mark off the figures in *groups* or *periods of three* towards the left. We shall then have four groups, the fourth, or last, containing only two figures: 93,428,965,347. The first period is evidently three hundred and forty-seven *units;* the second period is nine hundred and sixty-five *thousands;* the third period is four hundred and twenty-eight *millions;* and the fourth, or last, period is ninety-three *billions.* Let us always remember that the order of the *periods* is: *units, thousands, millions, billions, trillions, quadrillions, quintillions,* etc., every period getting a new name, *sextillions, septillions, octillions,* etc. Keeping this in mind, we can readily numerate our number of eleven figures as follows: 93 billions, 428 millions, 965 thousands, and 347 units, or rather 347, the word *units* being seldom used unless for the sake of special distinction.

19. *Now, before attempting Notation, perform at sight the following questions in Numeration.*

PRACTICAL QUESTIONS. No. 2.

ARABIC NUMERATION.

4. *Read off at sight the following:*

87632, 594376, 1002, 5980040, 3296873, 1000001, 3040506070, 37643098, 63092003961, 3296748164. (Repeat this exercise until you can numerate each quantity with great facility.)

More blackboard exercise may be necessary here, and should be given. For some pupils, smaller numbers might be best to begin with.

20. *Now tell me how to* notate, *that is, to express readily by figures any number that you may choose to ask me.*

I do so by giving you a simple example. Suppose you had to write *thirteen millions, twelve thousand, and sixty-*

four. First, you set down 13, and, since it is to be *millions,* it must lie in the third group, which you therefore denote thus: 13, , . Then you set down the 12, which, being *thousands,* must lie in the second group; thus: 13, 12, . Then set down the 64, which, being units, belongs to the first group. The number then stands as follows: 13, 12, 64; and as each group must contain three figures, we fill up the vacant spaces by noughts. This being done, and the whole work standing as follows, 13,012,064, we may, if we please, remove the marks separating the periods, there being no longer any danger of the figures getting into the wrong ranks.

21. *Give me another example.*

Suppose you were to *notate* one hundred and six *trillions,* seventeen *billions,* seven hundred and forty-six *thousands,* three hundred and eighteen (units). First write down 106, and, since it is to denote *trillions,* it is to stand at the left of the fourth mark, thus: 106, , , . Then put 17 to the left of the third or billion mark; there being no millions, you put nothing to the left of the second, or million, mark. Next write 746 at the left of the thousand mark, and 318 units in the first period. The whole work, standing as follows, 106, 17, ,746,318, can be evidently completed by filling up the vacant spaces by the necessary noughts: 106,017,000,746,318. We may then remove the marks, though in large numbers it is generally better to let them remain for the sake of easier Numeration.

22. *Say all this in a few words.*

First, write down the highest figures; second, make the necessary marks; third, set the other figures in their proper periods; fourth, complete by noughts whatever period or group has not three figures.

PRACTICAL QUESTIONS. No. 3.

ARABIC NOTATION.

Notate the following:

5. Twelve hundred and sixty-four.

6. Seventeen *thousand* and six.

7. Seven hundred *thousand* and twenty-four.

8. Four hundred and sixty-eight *thousand* and sixty-nine.

9. One hundred and twenty-seven *millions*, eight hundred and six *thousand*, and ninety-three.

10. Seventeen *billions*, thirteen *millions*, twelve *thousand* and sixty-eight.

11. Thirty-nine *trillions*, seventeen *millions*, seventy-four *thousand* and *ten*.

12. Fourteen *quintillions*, one hundred and sixty-six *millions* and seventeen.

13. Eight *trillions*, eight *millions*; eight *thousand* and eight.

14. Eleven *thousand* eleven *hundred* and eleven. (A catch.)

15. Thirteen *quadrillions*, seven *thousand* and seventy-four.

16. One hundred and twenty-three *trillions*, four hundred and fifty-six *millions*, seven hundred and eighty-nine.

CHAPTER II.

ADDITION.

23. *After learning how to Numerate and to Notate, what is the Arithmetic student to do?*

He must make himself a perfect master of the Four Rules, namely, Addition, Subtraction, Multiplication, and Division. In all of these he will be greatly aided by his knowledge of Notation and Numeration.

24. *What is Addition?*

By Addition we find the sum of two or more numbers denoting quantities of the same kind. For example: 6 dollars and 7 dollars amount to 13 dollars. This may be expressed more shortly by symbols: $6 + 7 = 13$. (Read *six plus seven equals thirteen.*)

An upright cross +, called *plus*, is the sign or symbol of Addition. Two horizontal lines =, called *equal*, is the sign or symbol of Equality. The amount of the quantities to be added is called their *Sum.*

Addition may be also considered as *counting forward;* example: $5 + 3 = 8$. Here 8 is the third number after 5 when we count forward, saying 6, 7, 8.

25. *Must the quantities of which we wish to find the sum be always of the same kind?*

Yes; we cannot express the sum of quantities of an unlike kind. It would be absurd to add together 3 peaches and 5 days, or 2 miles and 7 dollars. We should never forget that addition questions are possible only when the quantities are of exactly the same kind.

26. *How is Addition performed?*

When the numbers are small, we add them together in our minds and find their sum without much difficulty. For example: if we are asked what is the sum of 8 and 5 and 7, we are not long in telling that it is 20. If we are rather slow about it, we should practise this mental addition until we find ourselves tolerably ready and correct at the operation. Such readiness will assist us greatly in working at written Addition.

PRACTICAL QUESTIONS. No. 4.

MENTAL OR ORAL ADDITION.

(Not to be done with a Pencil.)

17. $5 + 8 + 7 = ?$ (Read: *Five plus eight plus seven equals how many?*)

18. $8 + 9 + 5 + 3 + 7 = ?$

Work of this kind, called *Plus Work*, should be done first *surely* and then *rapidly*. Quickness at Addition is the foundation of success in Arithmetic.

19. $4 + 6 + 7 + 3 + 9 + 8 + 6 + 5 = ?$
20. Take a number, 4 for example, and add 3 to it rapidly until you pass fifty. Thus: 4, 7, 10, 13, 16, 19, 22, etc.
21. Take 2 and add 6 to it continually until you pass a hundred.
22. Add 3 continually to 8 until you pass 107.
23. Add 4 continually to 9 until you pass 105.
24. Add 5 continually to 6 until you pass 104.
25. Add 6 continually to 7 until you pass 103.
26. Add 7 continually to 5 until you pass 101.
27. Add 8 continually to 4 until you pass 104.
28. Add 2 rapidly to 12 until you come to 100.
29. Add 3 rapidly to 3 until you come to 99.
30. Add 4 rapidly to 4 until you come to 100.
31. Add 5 rapidly to 5 until you come to 100.
32. Add 6 rapidly to 6 until you come to 96.
33. Add 7 rapidly to 7 until you come to 98.
34. Add 8 rapidly to 8 until you come to 96.
35. Add 9 rapidly to 9 until you come to 99.
36. Add 11 rapidly to 11 until you come to 110.
37. Add 12 rapidly to 12 until you come to 144.

All this shows that Addition is simply *counting forward* in jumps, always skipping the intervening figures.

27. *How is written Addition performed?*

In the first place, the numbers to be added must be set down properly; that is, the units must lie exactly under the units, the tens under the tens, the hundreds under the hundreds, and so on. Then, beginning with the unit column, you add up each column separately, placing units under units, tens under tens, hundreds under hundreds, etc., as before. If this does not give one continuous sum, repeat the operation on the new numbers until each figure is at last at home in its own place. Example: Suppose we had to add 764 + 372 + 1709 + 64 + 4263 + 37294. First set them down properly, as follows:

$$\begin{array}{r}
764 \\
372 \\
1709 \\
64 \\
4263 \\
37294 \\
\hline
\end{array}$$

The figures of the units column added together amount to 26, which we set down with the six under the units and the 2 under the tens. The tens column amounting to 34, you put the 4 in the tens column, passing the 3 to the hundred column, as the 34 tens are the same as 3 hundreds and 4 tens. The third, or hundred, column amounts to 21, that is, 2 thousand and 1 hundred; therefore we set the 1 among the hundreds and the 2 among the thousands. Then we add the thousand, or fourth, column up, and, finding it amounts to 12, we set the 2 in the thousand rank and the 1 in the ten thousand, or fifth, column. Finally, the fifth column having only one figure in it, we set this, the 3, down in its proper place.

$$\begin{array}{ccccc}
 & & 7 & 6 & 4 \\
 & & 3 & 7 & 2 \\
 & 1 & 7 & 0 & 9 \\
 & & & 6 & 4 \\
 & 4 & 2 & 6 & 3 \\
3 & 7 & 2 & 9 & 4 \\
\hline
 & & & 2 & 6 \\
 & & 3 & 4 & \\
 & 2 & 1 & & \\
1 & 2 & & & \\
3 & & & & \\
\hline
4 & 4 & 4 & 6 & 6
\end{array}$$

We then repeat the operation with the new columns, setting the sum of each column in its proper place as before. So we find 6 in the unit column; 4 + 2, or 6, in the ten column; 3 + 1, or 4, in the third, or hundred, column; 2 + 2, or 4, in the fourth; and 3 + 1, or 4, in the fifth, making in all 44466, that is, forty-four thousand, four hundred and sixty-six. The work then stands as above.

28. *Is all this trouble necessary?*

Yes, it is all necessary; but once that we thoroughly understand the management of the figures, we can save considerable time and trouble by what is called *carrying* certain figures in our heads, so that the second rows of columns need not be written out.

29. *What do you mean by* carrying *certain figures?*

I mean that it is enough to set down the *first* figure only if I add the *second* figure to the next column. This adding of the second figure to the next column without setting it down in its proper place, is what is meant by *carrying* it.

The principle of *carrying* is here explained, but it cannot be well understood until the Pupil is practised in adding by the double column.

30. *Give me an example of carrying.*

In the above example, instead of writing down 26, I write 6 only, adding 2, the ten figure, to the other figures in the ten column. These amounting to 36, I write down the 6 only, carrying the 3 to the next, or hundred, column. The hundreds being found to be 24, I write down the 4, and carry the 2 to the next column, which amounting to 14, I set down the 4, and carry 1 to the 3 in fifth column. This amounts to 4, so I find the sum to be as before, 44666.

31. *Which is the better way to add, by carrying or by the double column?*

The best way to *learn* to add is by double or even triple columns until you fully understand the principle of carrying; after that, no more double columns; the carrying method only should be employed.

PRACTICAL QUESTIONS. No. 5.

WRITTEN ADDITION.

(Model Operation.)

Add together six hundred and nineteen *thousand*, seven hundred and forty-four. Eighty-six *millions*, four hundred and twenty-eight *thousand*, five hundred and forty-three. Three hundred and sixteen *millions*, one hundred and five *thousand*, nine hundred and sixteen. Seventeen *millions*, nine hundred and eighty-seven *thousand*, six hundred and seventy-eight.

			6	1	9	7	4	4
	8	6	4	2	8	5	4	3
3	1	6	1	0	5	9	1	6
	1	7	9	8	7	6	7	8
							2	1
						1	6	
					2	7		
				2	9			
			1	1				
		2	0					
	1	9						
1	0							
3								
						8	8	1
				1	1			
	1	1	1	3				
4	1							
4	2	1	1	4	1	8	8	1

The next ten questions should be done after the above model. The pupils should even draw the lines, so that they may see the necessity of keeping each figure strictly in its own place. Thus they may learn the right meaning of *Carrying*, and the advantage of being able to *carry* readily and correctly.

ADDITION BY THE DOUBLE OR TREBLE COLUMNS.

38. Add six hundred and fifteen millions, four hundred and sixty-seven thousand, nine hundred and eighty-seven. Five hundred and sixty-seven millions, eight hundred and forty-eight thousand, nine hundred and sixty-nine. Eighteen hundred and seventy-six. Ninety-nine thousand, nine hundred and nine. Eight hundred and sixty-seven millions, five hundred and eighty-nine thousand, six hundred and ninety-seven.

39. 379876298 + 198756579 + 389784578 + 975864689 = ?

40. 5737867 + 2983769 + 874296 + 6894947 + 387891 + 763478 = ?

41. 39678432 + 7916873 + 29847 + 37896502 + 83279654 + 2387 = ?

42. 8163729 + 3968473 + 37916 + 291684 + 987654 = ?

43. 785432 + 694327 + 893587 + 916384 + 8976543 + 387 = ?

44. 567897 + 693576 + 934286 + 796873 + 296389 + 45849 = ?

45. 834296 + 383537 + 89684 + 659837 + 373589 + 37895 = ?

46. 987345678 + 789542678 + 589693468 + 764289758 + 579682357 = ?

47. 1264 + 17006 + 700024 + 468029 + 127806093 + 17013012068 = ?

The Pupil should be now ready for Addition by *Carrying.*

32. *Repeat what you mean by Carrying.*

By *carrying* I mean setting down the *right hand figure* only, and keeping the rest in my mind for addition to the next column. For example: if a column amounts to forty-nine, I set down the 9 only, carrying the 4 in my mind for the next column. Addition by *carrying* is not so easy as by the columns, but, being quite as correct and far rapider, it should be learned as soon as possible.

33. *Give an example of the two methods.*

The *Carrying* method is quicker and takes up much less room, as can be seen by the following examples of Addition:

BY THE DOUBLE COLUMNS.	BY CARRYING.
457896	457896
684387	684387
579124	579124
686579	686579
974382	974382
28	3382368
34	
20	
30	
35	
30	
3382368	

34. *Recite how the above sum is obtained by carrying.*

The first column amounting to 28, I set down the 8 only, carrying 2 to the second. The second column amounting to 36, I set down the 6, and carry the 3. The third column amounting to 23, I set down the 3, carrying the 2 as before. The fourth amounting to 32, the 2 is set down and the 3 carried. The fifth amounting to 38, the 8 is set down and the 3 carried. Finally, the sixth column amounting to 33, both figures are set down in their proper places, and the work is done.

35. *How am I to show that the sum is correct?*

Chiefly in two ways:

(1.) When the columns are neither long nor numerous, *count each column from down up, then from up down;* if the same result is obtained each way, the sum is extremely likely to be correct.

This simple proof should be always made.

(2.) When the sum of a considerable number of quantities is to be found, divide the work into two parts. First find the sum of the whole taken together; then find the sum of each part; lastly, adding the last sums together, if they are equal to the sum first obtained, it is almost impossible that you are not right.

36. *Give an example of the second way.*

478414896		
769284316		PROOF.
199642858		
255194		1447597264
416174318		
425475893		
555829721		
520129		1398000061
2845597325	=	2845597325

In the above, I first find the sum of the eight rows taken together, making sure I am right by seeing that the upward and downward additions of each column agree. Then separating the rows into two parts by a line, I find the sum of the first four rows to be 1447597264, and that of the second to be 1398000061. Adding these together, and finding the result to be exactly equal to the sum first obtained, I may be certain I am right.

37. *Is such a proof always necessary?*

Yes, such a proof is necessary in all tolerably large Additions. The Pupil is strongly recommended to form the habit of always proving his result; he is then on the sure way of becoming at once quick and accurate.

Proof by the Nines might be given here, but it is more useful in Multiplication and Division, for which it is deferred.

PRACTICAL QUESTIONS. No. 6.

ADDITION BY CARRYING.

Each example is to be proved as in *36.*

WRITTEN ADDITION.

48. Add seven hundred and fifty-nine thousand, eight hundred and seven + seven hundred and twenty-eight millions, four hundred and four thousand, eight hundred and sixteen + five hundred and sixty-three + eighteen hundred and eighty-six + three hundred and sixty-four thousand + thirteen thousand six hundred and seventy-four + twelve billions, six millions, five thousand and eight. (Prove.)

49. Add 199642908 + 255144 + 416174318 + 425475893 + 555829721 + 520129 + 478414896 + 769284316. (Prove.)

50. Add 31729 + 178296 + 36918742 + 16947396 + 1864 + 131296 + 98754321 + 168984 + 144256512 + 360004. (Prove.)

51. Add 21296 + 379284 + 165982 + 138649 + 596837 + 79642 + 161984 + 3694283. (Prove.)

52. Add 795 billions, 583 thousand, nine hundred and sixteen + nine hundred and twelve thousand, seven hundred and eighty-three + eight hundred and sixty-seven millions, four hundred and twenty-six thousand, eight hundred and ninety-eight. (Prove.)

CHAPTER III.

SUBTRACTION.

38. *What is Subtraction?*

Subtraction is the reverse of Addition. By Addition we find the sum of two quantities; by Subtraction, we find their difference. That 12 + 7 = 19 we find by Addition; that 12 — 7 = 5 we find by Subtraction. In Addition we count forward by a jump over the intervening figures, saying 14 + 8 = 22. By Subtraction we count backward, also jumping over the intervening figures, saying 14 — 8 = 6.

The sign (—), called *minus*, the sign of Subtraction, denotes how much is to be skipped as we count backward. Example: 17 — 4 means that in counting from 17 we jump backward to the fourth, which is 13; 17 + 4 means that in counting from 17 we jump forward to the fourth, which is 21.

As in Addition, so in Subtraction, the quantities operated upon must be of the same kind.

39. How is Subtraction performed?

When the numbers are small, we subtract the less from the greater in our minds without much difficulty. For example: if we are asked the difference between 5 and 8, we find it in one of two simple ways. We either count forward from 5, and we find that 8 is the third number; or we count backward from 8, and we find 5 to be the third number. Therefore the difference between these two numbers is 3.

We should practise this mental Subtraction until we are pretty expert at it.

PRACTICAL QUESTIONS. No. 7.

MENTAL SUBTRACTION.

53. From 28 take in succession 3, 5, 7, 9, 11, 14, 17, 21, and tell the remainders.

Answer as follows: 28 — 3 = 25; 28 — 5 = 23; 28 — 7 = 21; 28 — 9 = 19; 28 — 11 = 17; 28 — 14 = 14; 28 — — 17 = 11; 28 — 21 = 7.

54. From 59 take 2 rapidly until you come to 1.

Answer as follows: 59, 57, 55, 53, 51, 49, etc.

55. From 59 take 3 rapidly until you come to 2.

56. From 59 take 4 rapidly until you come to 3.

57. From 71 take 5 rapidly until you come to 1.

58. From 85 take 6 rapidly until you come to 1.

59. From 97 take 7 rapidly until you come to 6.

60. From 108 take 8 rapidly until you come to 4.
61. From 109 take 9 rapidly until you come to 1.
62. From 112 take 10 rapidly until you come to 2.
63. From 120 take 11 rapidly until you come to 10.
64. From 140 take 12 rapidly until you come to 8.

In all this the Pupil should be well exercised before going any farther.

40. *When the numbers are not small, how is Subtraction to be performed?*

Before this question can be answered satisfactorily, we must learn the meaning of some new names. The larger number from which the other is to be subtracted is called the *Minuend*, meaning what is to be diminished; the smaller number is called the *Subtrahend*, meaning what is to be subtracted; and their difference is called the *Remainder*. For example: if we have to take 234 from 687, 234 is the Subtrahend, 687 is the Minuend, and 453 is the Remainder.

In other words, it is before the Subtrahend that the sign *Minus* is to be placed, never before the Minuend.

41. *In setting down these quantities, must we treat them differently?*

Yes; we set them in two rows, the Minuend in the upper row, the Subtrahend in the lower, with units exactly under units, tens under tens, etc., as in Addition. Then subtracting, in each column, the lower from the upper figure, we set the difference down in its proper place, and the total Remainder is found.

42. *Give me an example in written Subtraction.*

Suppose I have to find the difference between 4532 and 37875. I set the greater quantity in the upper row, and the smaller in the lower, keeping the units under the units, the tens under the tens, etc. I then subtract the lower figure from the upper in each, 2 from 5, 3 from 7, 5 from 8, 4 from 7, and nought from 3. Setting the differences down in the proper columns, I have the total Remainder.

$$\begin{array}{r l}
3\,7\,8\,7\,5 & = \text{Minuend.}\\
\underline{4\,5\,3\,2} & = \text{Subtrahend.}\\
3\,3\,3\,4\,3 & = \text{Remainder.}
\end{array}$$

43. *Of what has the learner to be particularly careful in Subtraction?*

Of two things: to set the Minuend in the upper row; and to keep each figure in its own column. Heedless learners often get themselves into trouble by making such mistakes as the following:

BY WRONG ROWS.	BY WRONG COLUMNS.
4 5 3 2 Subtrahend.	3 7 8 7 5
3 7 8 7 5 Minuend.	4 5 3 2

Very little care, however, will keep us clear of such simple blunders.

44. *But when some of the lower figures are greater than those over them, what is to be done?*

The readiest answer to this question is given by a simple example. Suppose I have to subtract 286 from 654. I first set down the Minuend, and under it the Subtrahend as follows:

$$\begin{array}{r}
6\,5\,4\\
\underline{2\,8\,6}
\end{array}$$

Here it is evident that, though the lower number is smaller than the upper, the first two figures are greater than those over them, and, therefore, cannot be subtracted. But this difficulty we can readily overcome by remembering the great principle of the Arabic Notation. As long as the left figure of the Minuend is of greater value than the left figure of the Subtrahend, we can always subtract by means of *borrowing*.

45. *What do you mean by Borrowing?*

I answer by another example. Suppose a boy that has 2 dimes and 3 cents, owes 8 cents, how is he to pay his debt? By changing one of the dimes, he finds himself in possession of 13 cents, out of which he readily pays the 8 cents, having one dime and 5 cents left. Both cases of the question may be represented as follows:

FIRST CASE.

Out of 2 dimes + 3 cents.
Pay 8 cents.

Here the answer is not very clear.

SECOND CASE.

Out of 1 dime + 13 cents.
Pay 8 cents.
Remainder = 1 dime + 5 cents.

Here the answer is perfectly clear and simple.

46. *It is no doubt perfectly clear, but what has it to do with Borrowing?*

To make his subtraction possible, the boy had to take ten cents out of his dimes and add them to the 3, making 13 in all. This is what is meant by *borrowing* in Subtraction. When a figure is too small to be operated on, it is to be increased by 10, and this 10 is obtained by *borrowing* 1 from the next higher denomination. Doing so, evidently, does not change the total value of the number in any respect.

Borrowing may not be a good name for the operation, but a better one is hard to find, and, when once well understood, it can hardly be objected to.

47. *Now apply this borrowing principle to your example in Subtraction.*

We were brought to a stand-still by the figures as they lay, no Subtraction being possible:

$$\begin{array}{r}654\\ \underline{286}\end{array}$$

But by the *borrowing* principle every difficulty is at once cleared away. The 4 units we shall consider 14 units, by adding to it 1 ten taken out of the 5, which is thus reduced to 4. The 4 tens we shall consider 14, by adding to it one ten, taken out of the 6, which is thus reduced to 5. We set the new numbers over the old ones thus:

$$\begin{array}{ccc}(5) & (14) & (14)\\ 6 & 5 & 4\\ 2 & 8 & 6\\ \hline 3 & 6 & 8\end{array}$$

Then, neglecting the middle line altogether, and subtracting 6 units from 14 units we have 8 units, 8 tens from 14 tens we have 6 tens, and 2 hundred from 5 hundred we have 3 hundred, thus finding the total remainder to be 368.

It is evident that by making these changes in the figures we make no change in the real value of the Minuend, 5 hundred + 14 tens + 14 units amounting to exactly the same sum as 6 hundred, 5 tens, and 4 units.

$$\begin{array}{ll}\text{5 hundred} & = 500\\ \text{14 tens} & = 140\\ \text{14 units} & = \underline{14}\\ \text{Total} & = 654\end{array}$$

48. *Give me a few more examples in Subtraction.*

From 1237542 take 987853. Here the Subtrahend figures being all larger, I must arrange all the Minuend figures before attempting Subtraction.

$$\begin{array}{ccccccc}(0) & (11) & (12) & (16) & (14) & (13) & (12) &\\ 1 & 2 & 3 & 7 & 5 & 4 & 2 & = \text{Minuend.}\\ & 9 & 8 & 7 & 8 & 5 & 3 & = \text{Subtrahend.}\\ \hline & 2 & 4 & 9 & 6 & 8 & 9 & = \text{Remainder.}\end{array}$$

That once done, the work is easy.

From 9876875 take 3454354. Here the Subtrahend figures being all the smaller, no further arrangement is necessary.

9 8 7 6 8 7 5 = Minuend.
3 4 5 4 3 5 4 = Subtrahend.
6 4 2 2 5 2 1 = Remainder.

From 6792879 take 3684584. Here, only some of the Subtrahend figures being smaller, only a partial arrangement is necessary.

6	7	(8)	(12)	(7)	(17)	9
6	7	9	2	8	7	9
3	6	8	4	5	8	4
3	1	0	8	2	9	5

49. *What must I try to remember in arranging the figures?*

Unless the lower figure is greater than the one above, never to *borrow* from, that is, to diminish, the next column.

PRACTICAL QUESTIONS. No. 8.

WRITTEN SUBTRACTION.

(To be done by arranging the figures of the Minuend.)

50. As this operation requires some thought, the Teacher will take care that it is done correctly. The two rows being set down, the columns should be carefully inspected, one by one, beginning, of course, with the units. Whenever the lower figure is greater than the upper, the latter should be increased by 10, and the Minuend figure in the next column diminished by 1. If the figure to be diminished is 1 or 0, the alterations will be 10 or 9 respectively, and the second next figure is to be diminished by 1. Actual work will make all this very plain.

65. From 5476348 take 2765452.

MODEL OPERATION.

(4)	(14)	7	(5)	(12)	(14)	8	
5	4	7	6	3	4	8	= Minuend.
2	7	6	5	4	5	2	= Subtrahend.
2	7	1	0	8	9	6	= Remainder.

66. From 1764327 take 475216.
67. From 645983 take 494582.
68. From 76104327 take 59438728.

$$\begin{array}{cccccccc l} (6) & (15) & (10) & (9) & (13) & (12) & (11) & (17) & \\ 7 & 6 & 1 & 0 & 4 & 3 & 2 & 7 & = \text{Minuend.} \\ 5 & 9 & 4 & 3 & 8 & 7 & 2 & 8 & = \text{Subtrahend.} \\ \hline 1 & 6 & 6 & 6 & 5 & 5 & 9 & 9 & = \text{Remainder.} \end{array}$$

This is an illustration of the latter part of Remark 50.

69. From 4831704 take 398789.
70. From 6380176 take 4219854.
71. From 1239874 take 986548.
72. From 6905136 take 5978362.
73. From 89217839 take 76542398.
74. From 4878971 take 3967584.
75. From 18965324 take 9878657.

The Pupil should be now ready for Subtraction without arrangement of the Minuend.

51. *Is it always necessary to arrange beforehand the Minuend figures?*

No; doing so would be often very inconvenient and troublesome. Once, therefore, that we thoroughly understand the necessity of arranging them, and the art of doing so correctly, we can save ourselves much time and labor by arranging them in our minds.

52. *Explain what you mean by arranging the Minuend figure in your mind.*

Take the last example:

$$\begin{array}{r} 18965324 \\ 9878657 \\ \hline 9086667 \end{array}$$

Instead of making any arrangement of the upper line, we may go to work at once by saying: 7 from 4 we cannot; borrowing 1 ten from the 2 makes it 14; 7 from 14 leaves 7; set the 7 in the units. 5 from 1 we cannot; borrowing 1 from the 3 makes it 11; 5 from 11 leaves 6; set down 6 in the second column. 6 from 2 we cannot; borrowing 1 from 5 makes it 12; 6 from 12 leaves 6; set down 6 in the third column. And so on. In this way, after a little practice, we can find the Remainder quite as correctly as by the other method, and much more readily as well as more easily.

53. *Why not learn the easy way at once?*

Because in the last way, unless we are very careful, we are sure to make a mistake. If we forget to diminish the left hand figure from which we have borrowed we cannot help going wrong. It was to keep us on our guard, by showing us why we must diminish the left hand figure, that we learned the first way.

54. *How is a Subtraction question proved?*

We add the Remainder to the Subtrahend; if the sum is equal to the Minuend, we must be right.

PRACTICAL QUESTIONS. No. 9.

WRITTEN SUBTRACTION.

(To be done without arranging the figures of the Minuend.)

76. From 152847396 take 87564358. (Prove.)

MODEL OPERATION.

$$\begin{array}{r l}
1\,5\,2\,8\,4\,7\,3\,9\,6 & = \text{Minuend.} \\
8\,7\,5\,6\,4\,3\,5\,8 & = \text{Subtrahend.} \\
\hline
6\,5\,2\,8\,3\,0\,3\,8 & = \text{Remainder.} \\
\hline
1\,5\,2\,8\,4\,7\,3\,9\,6 & = \text{Minuend. Proof.}
\end{array}$$

In writing out the work, the Models should be strictly imitated.

77. From 379823654 take 28738956. (Prove.)

78. From 169025387 take 96578498. (Prove.)

79. From 300000122 take 29876543. (Prove.)

80. From one billion take one million, one thousand. (Prove.)

81. From two trillions take 123456789. (Prove.)

82. From five thousand and eighteen take eighteen hundred and seventy-six. (Prove.)

83. From 654837292 take 79587693. (Prove.)

84. From 1319674584 take 159175386. (Prove.)

85. From 2639349168 take 308340779. (Prove.)

PRACTICAL QUESTIONS. No. 10.

INVOLVING ADDITION AND SUBTRACTION.

86. From the sum of five *millions* and six, + thirteen thousand and thirty-eight, take the sum of one hundred and forty-seven *thousand*, nine hundred and forty-two, + eighty-nine *thousand* six hundred and fifty-three, + seventeen thousand, nine hundred and sixty-four.

MODEL.

	147,942	5,013,004	= Minuend.
5,000,006	89,653	255,559	= Subtrahend.
13,038	17,964	4,757,445	= Remainder.
5,013,044	255,559	5,013,004	= Proof.

87. Find the difference between 123456789 and 30004622. (Prove.)

Don't be so heedless as to try to subtract the greater number from the smaller.

88. Find the sum of the sum and difference of 217964 and 1389649. (Prove.)

89. 17296478 is the sum of two numbers, one of which is 8746927. What is the other? (Prove.)

When the pupil is asked a question about large numbers, let him ask himself *the same kind of a question* with small numbers. This will generally bring the first question so far within his comprehension that he will find little difficulty in mastering it. For example: before attempting the 89th, let him ask himself the following: 12 is the sum of two numbers of which one is 7, what is the other? 5. How is the 5 got? By subtracting one of the numbers from the sum of both. The 89th should then present no difficulty.

90. 13963845 is the Minuend; 57296 is the Remainder. What is the Subtrahend? (Prove.)

91. 896429 is the difference between two numbers, the greater of which is one million. What is the other number? (Prove.)

Imagine a little question like the following:

The greater of two numbers = 13
The smaller = ?
The difference = 7

What is the smaller here? How is it got? The smaller in the 91st is found in the same way.

92. How many times can 121728 be subtracted from 2 millions? What is left after the last subtraction? (Prove your work.)

93. How many times must 45639 be added to 99 to make the total sum equal to 365211?

94. A man has one thousand gold dollars to divide among sailors, but is to give them only 34 dollars apiece. How many sailors receive the money? How much is left over?

No Division to be allowed here.

PLUS WORK.

95. 2359 — 825 + 1625 — 1768 + 1096 = ?

The best way to do such a question is to add all the *Pluses*, then all the *Minuses*. Find the difference between the sums, and set it down with the proper sign. See Model on next page.

When no sign is before a quantity, + is always understood; example: 2359 = + 2359.

96. — 12096 + 17674 — 3869 + 130078 — 1574 = ?

97. 84963 + 38 — 17637 + 5742 — 11798 = ?

MODEL.

$$\begin{array}{rrl} & 84963 & \\ & +\quad 38 & -17637 \\ & +\ 5742 & -11798 \\ \cline{2-3} \text{Sum of } +\text{'s} & +90743 & -29435 = \text{Sum of } -\text{'s.} \end{array}$$

$$\begin{array}{rl} 9\,0\,7\,4\,3 & = \text{Minuend.} \\ 2\,9\,4\,3\,5 & = \text{Subtrahend.} \\ \hline 6\,1\,3\,0\,8 & = \text{Remainder.} \\ \hline 9\,0\,7\,4\,3 & = \text{Proof.} \end{array}$$

Ans. + 61308.

(It has the sign +, because the + sum is greater than the — sum.)

98. 89 — 981 + 22 — 78 + 875 — 1896 + 30076 — 1642 = ?

99. — 1684 + 17284 — 35964 + 17397 — 7851 + 4600 = ?

100. — 2908 + 3476 — 1632 + 19243 — 1264 + 137846 — 9987 + 133682 = ?

For additional questions in Addition and Subtraction, see Appendix at end of book.

CHAPTER IV.

MULTIPLICATION.

55. *What is Multiplication?*

Multiplication is a short and easy way to perform long and difficult operations in Addition.

56. *Why not learn it at once, instead of devoting so much time to Addition?*

Because Addition is always useful, whereas Multiplication can be employed only when the *same* quantity is to be added several times together. For example: If we had 30 bundles of books, each bundle containing a different number of books, the only way to find the total would be by actual addition. But if each bundle contained the *same* number of books, say 29, we shorten the work considerably by multiplying 29 by 30, finding almost in a second the total to be 870.

57. *Give another example of the use of Multiplication.*

I give an example of the same kind, only I change the numbers. Suppose I am asked the sum total of five different numbers; there is no shortening of the work possible. They must be all set down properly, and added up, as we have learned in Addition. But if I am asked to find how many dollars are in five boxes, each containing seven hundred and sixty-five thousand four hundred and thirty-eight dollars, I shorten even this simple task considerably by Multiplication.

58. *Explain how the work is shortened.*

If I did not know Multiplication, I should solve the question by Addition, as follows:

$$\begin{array}{r} 765438 \\ 765438 \\ 765438 \\ 765438 \\ 765438 \\ \hline 3827190 \end{array}$$

That is, I should set the number down *five times* in order to add it up. But by Multiplication I set the number down only *once*, and find the total even more readily. For example:

$$\begin{array}{r} 765438 \\ 5 \\ \hline 3827190 \end{array}$$

59. *Why do you set the figure 5 under the number to be added?*

Under the number to be added I set the number telling me *how many times* it is to be added. If it was to be added eight times, I should set 8 under it; if fifteen times, I should set 15 under it; and so on.

60. *How do you then proceed?*

I will tell you only a little now, for you will understand my answer better after awhile. I begin by saying, *5 eights are 40*, setting down the 0 and carrying 4. Then *5 threes are 15*, which, with 4 to be carried, makes 19; I set down 9 and carry 1. *Five times 4 are 20;* this, with the 1 carried = 21, 1 to be set down and 2 to be carried. This is all I need say for the present; but you see, no doubt, that setting down the result is done exactly as it was done in Addition.

61. *What names are given to the different terms concerned in Multiplication?*

The number to be repeated several times is called the Multiplicand; in above example 765438 is the Multiplicand. The number denoting how many times the Multiplicand is to be repeated is called the Multiplier; in above example 5 is the Multiplier. The result obtained by Multiplication is called the Product; in above, 3827190 is the Product.

62. *Are there any other names given to these terms?*

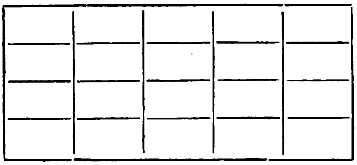
Yes; the Multiplier and the Multiplicand are also often called the Factors, and the Product is often called the Multiple of the Factors. An oblique cross (×) between the Factors is the sign of Multiplication; $5 \times 9 = 45$ is read: *Five multiplied by Nine equals Forty-five.* Five and nine are the Factors of forty-five, and 45 is the Multiple of 5 and 9.

63. *When speaking of two Factors, which is the Multiplier and which the Multiplicand?*

Of the two Factors it is indifferent which we consider the Multiplier and which the Multiplicand; five times nine must give the same result as nine times five.

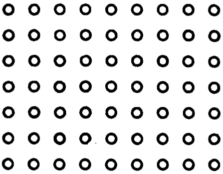
64.* *Why must the Product of two Factors be always the same, whichever one is taken as Multiplier?

A little illustration makes the answer to this question perfectly clear. Suppose the following diagram to represent a window-frame:



If we look at it lengthways, we see 4 rows, each containing 5 panes; that is, 4 × 5 = 20 panes in all. If we look at it breadthways, we see 5 rows, each containing 4 panes; that is, 5 × 4 = 20 panes, as before. However we look at it, that is, whether we multiply 5 by 4 or 4 by 5, the panes never change their number. Therefore, 5 × 4 and 4 × 5 must always give the same product.

Another example. If we have nine rows of marbles, containing seven in each row (as in the following diagram), we must have also seven rows of marbles containing nine in each row; that is, sixty-three always, however we count them.



Therefore, the product of two numbers is always the same, whichever is taken as multiplier.

Any other numbers may be taken as well as 4 and 5, or 9 and 7.

65.* *In order to multiply readily, what must we know well?

In order to multiply with readiness and correctness we should know perfectly what is called

THE MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

This is often called the Short Multiplication Table, to distinguish it from another which goes to 25 times.

66. *Explain this Table.*

It contains twelve *rows* running right and left, and twelve *columns* running up and down. The upper row shows the Multiplicands, and the left hand column the Multipliers. The Product of any two numbers is found in the square where the column of one meets the row of the other. For example: How many are six times seven? Take the sixth row and the seventh column; in the square where they meet you find 42, the Product required.

67. *What is the use of this Table?*

It gives us at a glance, the more easily to get them by heart, the sum totals of the lower numbers, as far as 12, when taken a certain number of times. For example: The first row contains the first twelve numbers; the second shows their doubles; the third, their triples; the fourth, four times as much; the fifth, five times as much; and so on to the twelfth, which contains twelve times as much. By getting each row by heart, therefore, we can tell in a second the sum, say of six nines, or seven eights, or nine twelves, without the tedious operation of working these results out in our minds.

68. *How am I to get this Table by heart?*

We begin at the second row, saying: *Twice one are two; twice two are four; twice three are six; twice four are eight;* and so on to *twice twelve are twenty-four.* Then taking the third row, we say: *Three times one are three; three times two are six; three times three are nine; three times four are twelve,* etc., finishing at *three times twelve are thirty-six.* And so we proceed through the whole Table, knowing one row perfectly before beginning another, and proceeding no further until the Table is thoroughly committed to memory.

69. *How can I tell when I know the Multiplication Table perfectly?*

You know the Table perfectly when you can answer accurately and readily such questions as the following:

PRACTICAL QUESTIONS. No. 11.

THE MULTIPLICATION TABLE.

(To be answered orally.)

101. 3 times 9 = ? 5 times 8 = ? 7 times 6 = ? 9 × 12 = ? 5 times 11 = ? 12 × 8 = ? 11 × 9 = ? 6 × 8 = ? 7 × 12 = ? 9 × 4 = ? 11 × 11 = ? 12 × 7 = ? 12 × 12 = ? 8 × 7 = ?

102. Repeat as far as 24 the numbers of which 2 is a factor. As far as 36 those of which 3 is a factor.

103. Repeat as far as 48 those of which 4 is a factor. As far as 60 those of which 5 is a factor. Recite 2 times.

104. Recite as far as 72 the numbers of which 6 is a factor. As far as 84 the multiples of 7. Recite 3 times.

105. Recite in order the multiples of 8; of 9; of 10; of 11; of 12. Recite 4 times, backward.

106. How many 2's in 14? In 20? In 24? In 12? How many 3's in 24? In 18? 12? 33? Recite 5 times, backward.

107. How many 2's in 18? 3's in 21? 4's in 28? 5's in 40? 6's in 42? 7's in 84? 8's in 72? Recite 6 times, backward.

108. How many 9's in 54? In 108? 63? 99? How many 7's in 42? 56? 84? 49? 70? Recite 7 times, forward and backward.

109. How many 8's in 56? 9's in 54? 11's in 121? In 132? 12's in 36? 60? 96? 72? 132? 84? 108?

110. Name 3 sets of factors for 24. (*Ans.* 2 × 12, 3 × 8, 4 × 6.) Three sets for 36. Two sets for 48. Two sets for 72. What number is found 6 times in the Table (excluding Multiplicands and Multipliers)? What number is found 5 times? What nine numbers are found only once?

Keeping the Pupil at the foregoing and similar questions until he can answer with tolerable readiness is an excellent way to teach him the relations of the lower numbers. Ready and accurate knowledge cannot be too much insisted upon; it will save much time and labor hereafter.

70. *Now explain how I am to make a practical use of the Multiplication Table.*

Supposing I have to double 596487, that is, to multiply it by 2, I set down the Multiplicand first, and then the Multiplier, unit under unit, as follows:

$$\begin{array}{r@{\;=\;}l} 596487 & \text{Multiplicand.} \\ \underline{\quad\quad 2} & \text{Multiplier.} \end{array}$$

I then say *twice 7 = 14*, which I set down, as already done in Addition, the unit in the first column, and the ten in the second. Then I say *twice 8 = 16*, and set down the 6, as before, in the second column, and the 1 in the third. In fact, the rest of the work is Addition over again, and therefore requires no further explanation than the diagram:

	5	9	6	4	8	7	= Multiplicand.
						2	= Multiplier.
					1	4	
				1	6		
		1	2	8			
	1	8					
1	0						
1	1	9	2	9	7	4	= Product.

71. *This is clear enough so far, but suppose I had to multiply by 3?*

I proceed just as before, only instead of saying *twice 7 = 14*, I say 3 times 7 = 21, etc.

	5	9	6	4	8	7	= Multiplicand.
						3	= Multiplier.
					2	1	
				2	4		
			1	2			
		1	8				
	2	7					
1	5						
1	7	8	9	4	6	1	= Product.

72. *Now multiply the same number by 9.*

	5	9	6	4	8	7	= Multiplicand.
						9	= Multiplier.
					6	3	
				7	2		
			3	6			
		5	4				
	8	1					
4	5						
			1	3	8	3	
1	3	6	7				
4							
5	3	6	8	3	8	3	= Product.

The Pupil sees here that carrying is just as necessary in Multiplication as in Addition; nevertheless he may put off carrying a little while longer. The following questions should be done after the above models, the Pupil drawing the lines on his slate so as to keep each figure strictly in its place.

PRACTICAL QUESTIONS. No. 12.

MULTIPLICATION.

111. Multiply 7896543 by 4.
112. Multiply 3874156 by 5.
113. Multiply 7329187 by 6.
114. Multiply 6529374 by 7.
115. Multiply 8739153 by 8.
116. Multiply 1768327 by 9.
117. Multiply 5329584 by 10.
118. Multiply 3920654 by 11.
119. Multiply 1532986 by 12.
120. Multiply 8976543 by 12.

73. *How am I to know if the answers I have found to the above arithmetical questions are right?*

If the work is carefully done, the answer must be right. Still, as we are always liable to error, we can never be *certain* that the result is correct until we can prove it to be so beyond all doubt. The best possible proof is obtained by *reversing* the operation. That is, for example, if we find by one operation that 7 times 96 = 672, and if we then find by another operation that the seventh part of 672 is 96, we may rest certain that we are right. This reverse of Multiplication is called Division. Multiplication and Division, indeed, are so closely connected that we cannot fully understand the one without just as fully understanding the other. Dropping Multiplication, therefore, for the present, we take up Division.

CHAPTER V.

DIVISION.

74. *What is Division?*

Multiplication being shortened Addition when the same quantity is to be added a number of times, Division may be considered shortened Subtraction when the same number is to be subtracted a number of times. Also, as Multiplication enables us to find quickly double, treble, four times, five times, etc., any number, so does Division enable us to find quickly the half, the third, the fourth, the fifth, etc., of any number.

75. *Illustrate by an example.*

In the last chapter Multiplication showed us that 1789461 is 3 times 596487; Division can show us that the smaller number is exactly the third part of the greater. But before we can understand this fully, we must make a new use of the Multiplication Table. By treating it a little differently we can turn it into a Division Table.

76. *Show how the Multiplication Table may become a Division Table.*

Knowing that twice 6 = 12, we also know that the half of 12 must be 6; so the half of 18 must be 9, and the half of 24 must be 12. Also, as 3 times 7 = 21, the third part of 21 must be 7; the third part of 18 must be 6, and the third part of 36 must be 12. In the same way the fourth part of 20 is 5; of 28, 7; and of 48, 12. So on with the other numbers, only in order to answer such questions readily we must get the Multiplication Table by heart in a new way.

77. *In what new way must we get the Table by heart?*

Instead of naming a product, we name its two factors. For example: Taking any row, we divide each of its numbers by its first number as one factor, and of course we get each number in the top row as the other factor. Begin with the second row; its first number being 2, we divide every number in the row by 2, saying *2 into 2, once; 2 into 4, twice; 2 into 6, 3 times; 2 into 8, 4 times; 2 into 10, 5 times,* etc. Then, taking the third row, we say *3 into 3, once; 3 into 6, twice; 3 into 9, 3 times; 3 into 12, 4 times;* and so on to the end, when we say *3 into 36, 12 times.* Then we take the other rows in succession, and go through them in a similar manner.

PRACTICAL QUESTIONS. No. 13.

DIVISION BY THE MULTIPLICATION TABLE.

(To be answered orally.)

121. Divide 2 into all its products as far as 24.
122. Divide 3 into all its products as far as 36.
123. Divide 4 into all its products as far as 48.
124. Divide 5 into all its products as far as 60.
125. Divide 6 into all its products as far as 72.
126. Divide 7 into all its products as far as 84.
127. Divide 8 into all its products as far as 96.
128. Divide 9 into all its products as far as 108.

129. Divide 11 into all its products as far as 132.

130. Divide 12 into all its products as far as 144.

78. *Now answer readily such questions as the following:*

5 into 45? (*Ans.* 5 into 45 nine times, because 9 times 5 = 45.) 6 is contained in 42 how many times? (*Ans.* 6 is contained in 42 seven times, because 7 times 6 = 42.) 8 into 88? 7 into 84? 9 into 63? 12 into 60? 11 into 132? 5 into 60? 3 is contained in 36 how often? 4 in 32? 6 in 72? 8 in 72? 12 in 72? 8 in 96? 12 in 132? 3 in 27? 7 in 49? 7 in 63? 8 in 56? 9 in 72? The half of 24? 36? 44? 60? etc.

79. *Before undertaking slate work in Division, I wish to know if any new names are given to its various terms.*

Division being different from Multiplication, its terms have different names—the Divisor, the Dividend, and the Quotient. The Divisor is the number divided into the Dividend, and the Quotient is the number telling how many times the Divisor is contained in the Dividend. When we say *2 into 14 goes 7 times*, 2 is the Divisor, 14 is the Dividend, and 7 is the Quotient. Or when we say *60 divided by 12 = 5*, 60 is the Dividend, 12 the Divisor, and 5 the Quotient.

The Product evidently corresponds with the Dividend, and the two Factors with the Divisor and the Quotient.

80. *What is the sign of Division?*

The usual sign of Division is a horizontal line with a dot on each side (÷), and when met is read *divided by.* For example: $32 \div 4 = 8$ is read *thirty-two divided by four equals eight.* With this sign the Dividend always goes first. Division is also expressed by means of the horizontal line alone, the Dividend and the Divisor taking the place of the dots. For example: $\frac{56}{8} = 7$ is read *fifty-six divided by eight equals seven.* In this case the Dividend is always above the line.

81. *Now let me see how a Division question is done on the slate.*

I begin by showing how Division proves the correctness of an operation in Multiplication. Suppose the question to be:

Multiply 89476 by 2, and prove the operation to be right. I first give the Model, and then the explanation:

MODEL EXAMPLE.

	8	9	4	7	6	= Multiplicand.
					2	= Multiplier.
				1	2	
			1	4		
	1	8	8			
1	6					
1	7	8	9	5	2	= Product.

PROOF.

Divisor = 2	1	7	8	9	5	2	= Dividend.
	1	6					89476 = Quotient.
		1	8				
		1	8				
			0	9			
				8			
				1	5		
				1	4		
					1	2	
					1	2	

The Quotient being equal to the Multiplicand, the operation must be right.

EXPLANATION.—The Multiplication part being already understood, 178952 is readily seen to be the double of 89476. In order to prove that I am right, I wish now to show that the latter number is exactly the half of the former. This is done by Division, the operation being as follows: I first set down the Product that is to be tested, only I now call it a Dividend, as it is to be divided. To the left, separated by a short vertical line, I set 2 the Divisor, the former Multiplier. To the right of the Dividend I mark off another place for the Multiplicand, now to be called the Quotient. I then begin the work by saying *2 into 1 goes nought times;* but, as a nought on the left side of a number is useless, I set nothing at all in the Quotient place. I then say: *2 into 17 goes 8 times,* and set the 8 in the Quotient. It does not go exactly 8 times, but 8 is the nearest number I can get, 9 times being too much. Multiplying the 8 by 2, I set the product 16 under the 17, taking good care that the figures are strictly in line with each other. This I do by making those on the left agree, knowing that if the 1 is under the 1, all the others will keep their proper place. It would be a great blunder, for example, to put the 1 under the 7, because they do not belong to the same order of figures, as you will easily see yourself when more advanced. Drawing a horizontal line, I subtract 16 from 17, and set the difference 1 in its proper place, under the 6, and bringing down 8, the next figure in the Dividend, I have 18 to be divided by 2. Into this it goes 9 times; setting the 9 in the Quotient, I set the product of 9 times 2, or 18, under the 18, and draw a little horizontal line, as before, for subtraction. 18 from 18 leaving nothing, I may, if I like, set the noughts down in their proper places, but I don't do so, as noughts on the extreme left are useless, having there no effect whatever on the other figures. 9, the next figure in the Dividend, being then brought down, and 2 going into it 4 times, I set 4 in the Quotient, and set the product of 4 times 2, or 8, under the 9, to be subtracted as before, drawing as usual the short horizontal line. 8 from 9 leaving 1, I set down the 1, and, bringing down 5, I have 15 for the new dividend. Into this the 2 going 7 times, 7 is set in the Quotient; 7 × 2, or 14, is set under the 15; the line is drawn; the remainder 1 is set down in the proper place, and 2 being brought down, we have 12 still to be divided by 2. Into this the 2 going 6 times, the 6 is set in the Quotient; the proper multiplication is made and set in its place, and the line is drawn; but as nothing is found to be remaining, and as all the figures are now brought down, the line is made double, and the work is done. It is also right, for the Quotient, the former Multiplicand, is shown to be exactly the half of the Dividend, the former Product.

The Pupil should go no further until he can explain every part of the foregoing, bit by bit. He should then try to explain, almost in the same words, each step in the following example. If tolerably successful in doing so without help, he is pretty well prepared to undertake work of his own.

ANOTHER MODEL.

Multiply 589746 by 3, and prove by Division.

	5	8	9	7	4	6	= Multiplicand.
						3	= Multiplier.
					1	8	
				1	2		
			2	1			
		2	7				
	2	4					
1	5						
1	7	6	9	2	3	8	= Product.

PROOF.

Divisor = 3	1	7	6	9	2	3	8	= Dividend.
	1	5						589746 = Quotient.
		2	6					
		2	4					
			2	9				
			2	7				
				2	2			
				2	1			
					1	3		
					1	2		
						1	8	
						1	8	

PRACTICAL QUESTIONS. No. 14.

IN MULTIPLICATION AND DIVISION.

The following questions should be done strictly after the above Models, the lines being drawn on the slate by the Pupil himself. No carrying should be allowed for the present. Setting down *all* the figures shows the advantage of carrying and the necessity of carrying accurately.

131. Multiply 357968 by 4, and prove by Division.
132. Multiply 357968 by 5, and prove by Division.
133. Multiply 357968 by 6, and prove by Division.
134. Multiply 357968 by 7, and prove by Division.
135. Multiply 357968 by 8, and prove by Division.
136. Multiply 357968 by 9, and prove by Division.
137. Multiply 357968 by 10, and prove by Division.
138. Multiply 357968 by 11, and prove by Division.
139. Multiply 357968 by 12, and prove by Division.
140. Multiply 2468957 by 9, and prove by Division.

82. *Must you always begin at the left hand side in Division?*

Yes; to do the work otherwise is possible, but extremely difficult. In Multiplication we begin at the right, that is, with the low numbers, and, by carrying, advance to the left, that is, to the high numbers. In Division we begin at the left, that is, among the high numbers, and, by subtraction, go down to the right, that is, to the low numbers.

83. *A Division question, therefore, can be proved by Multiplication?*

Certainly, by reversing the operation. If the ninth part of, say, 144 is 16, nine times 16 must be 144. That is, nine times the Quotient must be the Dividend. To prove Division, therefore, we multiply Quotient by Divisor; if the Product equals the Dividend we must be right.

MODEL EXAMPLE.

Divide 847345761 by 9, and prove by Multiplication.

Divisor = 9	8	4	7	3	4	5	7	6	1	= Dividend.
	8	1								94149529 = Quotient.
		3	7							
		3	6							
			1	3						
				9						
				4	4					
				3	6					
					8	5				
					8	1				
						4	7			
						4	5			
							2	6		
							1	8		
								8	1	
								8	1	

PROOF.

		9	4	1	4	9	5	2	9	= Multiplicand.
									9	= Multiplier.
								8	1	
							1	8		
						4	5			
					8	1				
				3	6					
		3	6	9						
	8	1								
							1	6	1	
				1	4	5	6			
			1	2						
	8	4	6							
Dividend =	8	4	7	3	4	5	7	6	1	= Product.

The Product being equal to the Dividend, we are right.

No doubt some trouble is necessary to draw these lines, and some time seems to be wasted in drawing them properly, but no trouble is too great and no time is really wasted when we are learning the supreme value of *accuracy.*

PRACTICAL QUESTIONS. No. 15.

DIVISION AND MULTIPLICATION.

141. Find the half of 748965960, and prove by Multiplication.

142. Find the third of 748965960, and prove by Multiplication.

143. Find the fourth of 748965960, and prove by Multiplication.

144. Find the fifth of 748965960, and prove by Multiplication.

145. Find the sixth of 748965960, and prove by Multiplication.

146. Divide 1025640 by 7, and prove by Multiplication.

147. Divide 1025640 by 8, and prove by Multiplication.

148. Divide 1025640 by 9, and prove by Multiplication.

149. Divide 1025640 by 11, and prove by Multiplication.

150. Divide 1025640 by 12, and prove by Multiplication.

84. *Suppose, in the last example, that 12 did not go evenly into the Dividend, that is, that it left something over, how could you prove your work by Multiplication?*

Whenever there is an Over, the Divisor does not go evenly into the Dividend, and of course multiplying the Quotient by the Divisor will give us a Product not quite equal to the Dividend. But if to this Product we add the Over and then obtain the Dividend, we must be right. To prove, therefore, when there is an Over: add this Over to the Product of Quotient by Divisor. The Sum should be the Dividend.

Take the following example as a

MODEL.

Divide 35896 by 12, and prove by Multiplication.

Divisor = 12	3	5	8	9	6	= Dividend.
	2	4				2991 + 4 = Over.
	1	1	8			
	1	0	8			
		1	0	9		
		1	0	8		
				1	6	
				1	2	
					4	= Over.

PROOF.

Multiplicand =		2	9	9	1	+ 4
				1	2	= Multiplier.
					4	= Over.
				1	2	
		1	0	8		
	1	0	8			
	2	4				
Dividend =	3	5	8	9	6	= Product.

Explanation.—Here the 4 left over is seen to be annexed to the Quotient with the sign + before it, meaning that 12 is contained in the Dividend 2991 times, and that 4 remains over. In the proof this Over is set down in its proper place, the unit column, before the Multiplication begins, and then the operation goes on exactly as before.

PRACTICAL QUESTIONS. No. 16.

DIVISION PROVED WHERE THERE IS AN OVER.

151. Divide 4987651 by 4, and prove by Multiplication.
152. Divide 4987651 by 5, and prove by Multiplication.
153. Divide 4987651 by 6, and prove by Multiplication.
154. Divide 4987651 by 7, and prove by Multiplication.
155. Divide 4987651 by 8, and prove by Multiplication.
156. Divide 4987651 by 9, and prove by Multiplication.
157. Divide 4987651 by 10, and prove by Multiplication.
158. Divide 4987651 by 11, and prove by Multiplication.
159. Divide 4987651 by 12, and prove by Multiplication.
160. Divide 9654037 by 12, and prove by Multiplication.

85. *In working out these questions, are the vertical lines always necessary?*

The vertical lines being drawn merely to aid us in keeping the figures in the proper ranks, when we can do so without their aid, of course they are no longer necessary. We are now going to learn how we can dispense with them altogether, and therefore considerably shorten our work. In Multiplication there is, however, little to be taught, the chief points being already learned in Addition, and even in Division the most of the difficulties are already overcome.

86. *In what respect is the work shortened?*

The vertical lines need not be drawn in the first place; then, by properly carrying in my head the number to be added or to be subtracted, the work can be done in one row. Take a Multiplication question for the first example. Suppose I have to multiply 54983 by 8. I set it down as before, and by properly carrying, as in Addition, the result appears almost as fast as it can be written.

$$\begin{array}{r l} 54983 & = \text{Multiplicand.} \\ 8 & = \text{Multiplier.} \\ \hline 439864 & = \text{Product.} \end{array}$$

***87.* Explain how this product is obtained so quickly.**

I begin by saying *8 times 3 = 24*, but instead of setting down 24 in full, I set the 4 in the unit column and carry the 2 in my head to add it to the product of the ten column. This being 8 × 8 = 64, I add the 2 to it, making it 66; the figure to the right I set down in the second column, carrying the other 6 for the third. 8 times 9 being 72, I add the 6, making it 78; the 8 or right hand figure I set down as before, carrying the 7 for the product of the next column. This being 32, and the 7 added making it 39, the 9 is set down and the 3 carried. Finally 8 times 5 being 40, and the 3 added making it 43, I set the whole 43 down, there being no other figure to multiply. The work is now done, the result appearing in one line, and set down as fast as it was obtained.

***88.* How is it proved to be right?**

By Division, which can also be done in one line. Suppose the question to be: Find the eighth part of 439864. I set the figures down properly, and after very little work the result appears in one line.

$$\text{Divisor} = 8\,\overline{)\underline{439864}} = \text{Dividend.}$$
$$54983 = \text{Quotient.}$$

***89.* Explain how this result is obtained in one line.**

After setting the Divisor to the left of the Dividend, separated from it by a little curved line, I draw another line under the Dividend, showing the place where the Quotient is to appear. Then, as already learned, 43 being the first dividend, I say *8 into 43 goes 5 times;* and set down the 5 under the 3. But instead of actually setting down 40, the product of 8 and 5, I subtract it in my mind from 43, and also in my mind I set the remainder 3 before the next figure, 9, thus making it 39, for the new dividend. This shows what I continue to do during the whole operation. I first discover the figure for the Quotient, and set it down in its proper place. Then making the multiplication in my mind, I subtract the product from the dividend, and find the new dividend by imagining the remainder to stand before the next figure. This being understood, I continue. 8 into 39 goes 4 times, set down the 4 and imagine the remainder 7 to stand before 8, making 78 the new dividend. 8 into 78 goes 9 times, with 6 over; setting down 9, and imagining the 6 to stand before 4, I find 64 to be the new dividend. 8 into 64 goes 8 times, set down the 8, and nothing more being left, the work is done. I find the Quotient to be 54983, which being the Multiplicand in the former question, both operations are proved to be right.

Here again the Teacher sees that the Pupil explains every step with tolerable correctness. The way is short only when it is correct. If not done with care, it is anything but short. Before being set to work at questions of his own, he should be able to explain each step in the following

MODEL.

Multiply 678947 by 9, and prove by Division.

$$\begin{array}{r l} 678947 & = \text{Multiplicand.} \\ 9 & = \text{Multiplier.} \\ \hline 6110523 & = \text{Product.} \end{array}$$

PROOF.

$$\text{Divisor} = 9\,)\underline{6110523} = \text{Dividend.}$$
$$678947 = \text{Quotient.}$$

PRACTICAL QUESTIONS. No. 17.

SHORT MULTIPLICATION AND DIVISION.

Either in Multiplication or Division, the inexperienced Pupil is likely to make a mistake in carrying. When he goes wrong in any of the following examples, he should be allowed to find out his mistake himself by doing the work *in the long way with the lines.* This will not only give him confidence in himself, but also teach him to be more careful.

161. Multiply 478956 by 5, and prove by Division.
162. Multiply 478956 by 6, and prove by Division.

163. Multiply 478956 by 7, and prove by Division.
164. Multiply 478956 by 8, and prove by Division.
165. Multiply 478956 by 9, and prove by Division.
166. Multiply 478956 by 10, and prove by Division.
167. Multiply 478956 by 11, and prove by Division.
168. Multiply 478956 by 12, and prove by Division.
169. Multiply 123456789 by 9, and prove by Division.
170. Multiply 987654321 by 9, and prove by Division.

90. *Can a Short Division question be proved by Short Multiplication?*

Yes, as before, by reversing the operation. Of course, however, if I am careless in the multiplication, the proof is good for nothing. Take the following

MODEL.

Divide 74456872 by 8, and prove by Multiplication.

$$\begin{array}{r l}
\text{Divisor} = 8\ )\underline{74456872} & = \text{Dividend.} \\
9307109 & = \text{Quotient.}
\end{array}$$

The Noughts must evidently be set down in order to keep the other figures in their proper place.

PROOF.

$$\begin{array}{r l}
9307109 & = \text{Multiplicand.} \\
\underline{8} & = \text{Multiplier.} \\
\text{Dividend} = 74456872 & = \text{Product.}
\end{array}$$

PRACTICAL QUESTIONS. No. 18.

SHORT DIVISION AND MULTIPLICATION.

171. Divide 1896345 by 5, and prove by Multiplication.
172. Divide 1896345 by 9, and prove by Multiplication.

In questions of this kind young pupils, *guessing* themselves to be right, are liable to set the proof down without actual multiplication. Such a state of mind is the greatest foe to all improvement in arithmetic. The

Pupil is taught carefulness by doing his work *the long way with the lines* when he is reckless in doing it the short way without them.

173. Divide 157369498 by 11, and prove by Multiplication.

174. Divide 157369498 by 7, and prove by Multiplication.

The Over is treated exactly as it was in the long way.

175. Divide 3742965 by 8, and prove by Multiplication.
176. Divide 3742965 by 9, and prove by Multiplication.
177. Divide 3742965 by 11, and prove by Multiplication.
178. Divide 39674283 by 9, and prove by Multiplication.
179. Divide 39674283 by 11, and prove by Multiplication.
180. Divide 39674283 by 12, and prove by Multiplication.

When the Pupil can solve the above with tolerable readiness and reasonable care, he is ready for Long Multiplication and Division.

CHAPTER VI.

LONG MULTIPLICATION AND DIVISION.

91. *What do you mean by Long Multiplication and Long Division?*

At Short Multiplication and Short Division, which we have just learned, the highest Multiplier or Divisor was 12. This is because the Multiplication Table that most people learn extends no further than *twelve times.* When the Multiplier or Divisor is greater than 12, we have recourse to Long Multiplication or Long Division. They are so called because more figures are necessary, neither Product nor Quotient being obtained at once in the first row of figures.

Some people are such skilful arithmeticians that they can multiply or divide in one line by almost any number up to a hundred. But most persons, finding this short way too difficult when dealing with a Multiplier or Divisor greater than 12, prefer to have recourse to the long way.

92. *Give example of an operation in Long Multiplication.*

Suppose 37964 is to be multiplied by 18. Not knowing 18 times, we cannot set down the product in one line. But as we know that 18 equals 8 units and 1 ten, we multiply first by the 8 units and then by the 1 ten. Finding the sum of the two products, we evidently have the total product. All we have to be careful about is to set down the products so correctly that they can be added up properly. This is easily done, as may be seen by the following example:

$$\begin{array}{rl} 37964 & = \text{Multiplicand.} \\ \underline{\quad 18} & = \text{Multiplier.} \\ 303712 & = \text{Product by 8.} \\ \underline{379640} & = \text{Product by 10.} \\ 683352 & = \text{Total Product.} \end{array}$$

93. *Give another example in Long Multiplication.*

Suppose 78956 is to be multiplied by 87.

$$\begin{array}{rl} 78956 & = \text{Multiplicand.} \\ \underline{\quad 87} & = \text{Multiplier.} \\ 552692 & = \text{Product by 7.} \\ \underline{6316480} & = \text{Product by 80.} \\ 6869172 & = \text{Total Product by 87.} \end{array}$$

94. *Here you multiply by 80 in one line. How can you do so since 80 times is not in the Multiplication Table?*

It is just as easy to multiply by 80 as by 8. You have noticed already that in order to multiply by 10 we simply annex a nought to the right of the number to be multiplied. For example: 6 multiplied by 10 = 60; 12 multiplied by 10 = 120; and 34 multiplied by 10 = 340. This must be so, because annexing a nought to the right sets the figures back to a higher column, and therefore makes them 10 times more valuable than they were before. To multiply any number, say 53, by 80, therefore, all we need do is to suppose a nought to be at the right of 53, making it 530; then multiplying 530 by 8, we have 4240 = the product of 53 by 80.

95. *Then it must be just as easy to multiply by 60 as by 6, and by 90 as by 9?*

Just as easy; the product of any number by 90 is the product of the same number by 9, with a nought annexed to the right: 64 × 9 = 576; 64 × 90 = 5760.

96. *Then to multiply by 900, I simply multiply by 9, and annex two noughts to the right?*

Precisely so, because annexing two noughts to the right puts the figures back two columns, thereby making them 100 times more valuable than they were before. In the same way, to multiply by 9000, we annex three noughts to the product; by 90000, we annex four noughts to the product, and so on, whatever may be the number of noughts. This principle makes multiplication by high numbers a very simple matter.

97. *Must the noughts be always set down?*

The noughts need not even be actually set down to the right. It will be enough if we *imagine* them to be set down, and shift the other figures accordingly. Such noughts, meaning nothing in themselves, are not pleasant figures to deal with, and should therefore be invariably avoided unless when absolutely necessary. They may be all omitted in Long Multiplication if we are careful as to the column in which we set the first figure of each product.

98. *Where am I always to set this first figure of the product?*

The first figure of the product is always set down in the column to which the multiplier belongs. That is, when you multiply by a *unit*, the first figure of the product must be in the *unit* column; when you multiply by a *ten*, the first figure must be in the *ten* column; when by a *hundred*, in the *hundred* column; when by a *thousand*, in the *thousand* column, and so on. By doing this, we may omit the troublesome noughts. The first figure is evidently the only one that needs watching; that being once in its proper place, the others take theirs without further trouble.

The Pupil can show that he understands the above by explaining every step in the following examples:

Multiply 468263 by 264.

$$\begin{array}{rl}
468263 & = \text{Multiplicand.} \\
264 & = \text{Multiplier.} \\ \hline
1873052 & = \text{Product by } 4 \\
2809578 & = \text{Product by } 60 \\
936526 & = \text{Product by } 200 \\ \hline
123621432 & = \text{Product by } 264
\end{array}$$

Multiply 78096 by 4075.

$$\begin{array}{rl}
78096 & = \text{Multiplicand.} \\
4075 & = \text{Multiplier.} \\ \hline
390480 & = \text{Product by } 5 \\
546672 & = \text{Product by } 70 \\
3123840 & = \text{Product by } 4000 \\ \hline
318241200 & = \text{Product by } 4075
\end{array}$$

Here there being no hundred in the Multiplier we set nought in the hundred column, and, the 4 being a thousand, we set down the first figure of 24 in the thousand column.

99. *How are Long Multiplication operations proved?*

They are proved by Long Division, which, having been partially learned already, requires but little additional explanation. We found (93) the product of 78956 by 87 to be 6869172. Let us test this product by Long Division.

We set the Dividend down as before, the Divisor to the left, and to the right a place for the Quotient. 6 or 68 being too small for the first dividend, we take 686, the first three figures, for one, and, saying *87 into 686 goes 7 times,* we set the 7 in the Quotient, and 609, the product of 87 × 7, under the 686, to be subtracted.

$$\begin{array}{rr|l}
\text{Divisor} = 87 & 6869172 & = \text{Dividend.} \\
7 \times 87 = & \underline{609} & \underline{78956 = \text{Quotient.}} \\
 & 779 & \\
8 \times 87 = & \underline{696} & \\
 & 831 & \\
9 \times 87 = & \underline{783} & \\
 & 487 & \\
5 \times 87 = & \underline{435} & \\
 & 522 & \\
6 \times 87 = & \underline{\underline{522}} &
\end{array}$$

EXPLANATION.—Here evidently the greatest trouble is to find the quotient figure. Often it can be done only by trial. Here, for example, I find 7 times to be right, the remainder 77 being less than the Divisor 87.

100. *What is done if the remainder is greater than the Divisor?*

The quotient figure is then too small, and I must increase it until the remainder is less than the Divisor; then I know I have the right figure.

EXPLANATION, continued.—To the 77 remaining I annex the 9 brought down from the Dividend, making the new dividend 779. Into this 87 goes 8 times, as I find 8 times 87 to be 696. Subtracting this product from 779, and to the remainder annexing 1, the next figure to be brought down, I find the new dividend to be 831. I then find a new quotient figure, multiply the Divisor by it, subtract the product, to the remainder bring down the next figure for a new dividend; and so I proceed until all the figures are brought down. Then the work is done and the Quotient found.

101. *In case the remainder with the figure annexed is too small to be a dividend, what is to be done?*

When the Divisor will not go into the dividend, I put a nought in the quotient, and annex the next figure for a new dividend. I must not forget to set the nought in the Quotient; otherwise the figures could not keep their proper place.

MODEL.

Multiply 314086 by 1283, and prove by Division.

			3	1	4	0	8	6	= Multiplicand.
					1	2	8	3	= Multiplier.
			9	4	2	2	5	8	= Product by 3
	2	5	1	2	6	8	8		= Product by 80
	6	2	8	1	7	2			= Product by 200
3	1	4	0	8	6				= Product by 1000
4	0	2	9	7	2	3	3	8	= Product by 1283

PROOF.

1283	4	0	2	9	7	2	3	3	8	= Dividend.
1283 × 3 =	3	8	4	9						314086 = Quotient.
		1	8	0	7					
1283 × 1 =		1	2	8	3					
			5	2	4	2				
1283 × 4 =			5	1	3	2				
				1	1	0	3	3		
1283 × 8 =				1	0	2	6	4		
						7	6	9	8	
1283 × 6 =						7	6	9	8	

Finding that, after bringing down the 3, I cannot divide 1283 into the Dividend, I set a nought in the Quotient and bring down the next figure for a new dividend. In case this was still too small, I would set another nought in the Quotient and bring down another figure.

PRACTICAL QUESTIONS. No. 19.

LONG MULTIPLICATION AND DIVISION.

The first five of the following questions should have the vertical lines all drawn, and the rest of the work should be done as in the above model. In the next five the vertical lines may be omitted; the Pupil should learn to keep his figures straight without their aid.

181. Multiply 25873 by 126, and prove by Division.
182. Multiply 25873 by 257, and prove by Division.
183. Multiply 25873 by 584, and prove by Division.
184. Multiply 25873 by 1587, and prove by Division.
185. Multiply 25873 by 3695, and prove by Division.

In the next five operations the vertical lines may be omitted, unless the Teacher still thinks them necessary.

186. Multiply 89735 by 342, and prove by Division.
187. Multiply 89735 by 856, and prove by Division.
188. Multiply 89735 by 987, and prove by Division.
189. Multiply 89735 by 1375, and prove by Division.
190. Multiply 89735 by 5867, and prove by Division.

102. *How are Long Division operations proved?*

Multiplication is a perfect proof of Division, one operation being the exact reverse of the other. The reason is extremely simple. If 7 is contained 9 times in 63, 9 times 7 must be 63. If 12 is contained 11 times in 132, of course 11 times 12 must be 132. That is, if Divisor is contained in Dividend a certain number of times, Divisor multiplied by this certain number of times must equal Dividend. Therefore, to prove Division, obtain Dividend by multiplying Quotient by Divisor, adding Over, if any, to Product.

Long Division operations are proved by Long Multiplication, as may be seen in the following

MODEL.

Divide 896 into 75893789, and prove by Multiplication.

Divisor = 896		7	5	8	9	3	7	8	9	= Dividend.
896 × 8	=	7	1	6	8					84702 + 797
		.	4	2	1	3				Quotient.
896 × 4		=	3	5	8	4				
				6	2	9	7			
896 × 7			=	6	2	7	2			
						2	5	8	9	
896 × 2					=	1	7	9	2	
							7	9	7	= Over.

896 × 84702 + 797 = 75893789 = Dividend.

PROOF.

			8	4	7	0	2		+ 797 = Multiplicand.
					8	9	6		= Multiplier.
		5	0	8	2	1	2	=	Product by 6
	7	6	2	3	1	8		=	Product by 90
6	7	7	6	1	6			=	Product by 800
7	5	8	9	2	9	9	2	=	Product by 896
					7	9	7	= Over.	
7	5	8	9	3	7	8	9	= Dividend.	

Instead of letting the addition of the *Over* be seen as above, many mathematicians prefer to add it during the multiplication. They do so this way: When multiplying by a unit they add the units; thus, in above they would say, *six times 2 = 12; + 5 = 17*, setting 7 in the unit column. In multiplying by 9 they say, *nine times 2 = 18; + 9 = 27*, setting the 7 in the ten column; and when multiplying by 8 they say, *eight times 2 = 16; + 7 = 23*, setting 3 in the hundred column, and carrying 2. This is certainly the best way, but as some experience is required in following it correctly, a young pupil may omit it for the present.

PRACTICAL QUESTIONS. No. 20.

LONG DIVISION AND MULTIPLICATION.

In the first five of the following the vertical lines should be drawn, as above. In the next five they may be omitted.

191. Divide 489653 by 89, and prove by Multiplication.

192. Divide 489653 by 274, and prove by Multiplication.

193. Divide 489653 by 482, and prove by Multiplication.

194. Divide 489653 by 1796, and prove by Multiplication.

195. Divide 489653 by 5942, and prove by Multiplication.

Henceforward omit the = work on the right side of Long Multiplication, and on the left side of Long Division.

196. Divide 384296 by 256, and prove by Multiplication.

197. Divide 384296 by 574, and prove by Multiplication.

198. Divide 384296 by 1356, and prove by Multiplication.

199. Divide 72864798 by 89753, and prove by Multiplication.

200. Divide 72864798 by 485637, and prove by Multiplication.

In the last two questions the vertical lines would be found very useful; still the Pupil henceforward should be encouraged to do without them.

PRACTICAL QUESTIONS. No. 21.

TESTING THE PUPIL'S GENERAL KNOWLEDGE.

201. Divide the product of 3456 and 7892 by their difference, and prove by Multiplication.

202. Divide the sum of eighteen thousand and sixty-seven + nineteen thousand two hundred and seventy-eight by their difference, and prove by Multiplication.

203. Divide the product of the nine digits by their sum, and prove by Multiplication. *Ans.* 8064.

204. If the Divisor is 376 and the Quotient 2984, what is the Dividend? (Prove.)

HINT.—If Divisor is 8 and Quotient 9, what is the Dividend?

$$\text{Divisor} = 8 \;\Big|\; \underline{\text{Dividend} = ?}$$
$$\qquad\qquad 9 = \text{Quotient.}$$

205. If the Divisor is 167, the Quotient 186, and the Over 47, what is the Dividend? (Prove.)

Simple as is this question, many a Pupil is puzzled by it. It is done as follows:

Multiplicand = 186 + 47
Multiplier = 167
1302
1116
186
31062
47
31109 must be the Dividend.

PROOF.

Divisor = 167 | 31109 = Dividend.
167 | 186 = Quotient.
1440
1336
1049
1002
47 = Over.

The Dividend must evidently be the product of the Divisor and the Quotient.

206. The Divisor is 345, the Quotient 268, and the Over 199; what is the Dividend? (Prove.)

207. The Product of two Factors is 1270630; the half of one of these Factors is 3129; what is the other Factor? (Prove.)

HINT.—The Product of two Factors is 42; 3 is the half of one of these Factors; what is the other Factor?

208. Divide the sum of ninety-two thousand three hundred and eighty-four + two millions two thousand and two by the difference between 7648 and 13237. (Prove by Multiplication.)

209. Multiply three hundred and two thousand six hundred

and nine by eight hundred and fourteen thousand five hundred and forty-six. (Prove by dividing by multiplier.)

210. Multiply forty-four thousand and sixty-seven by seventeen thousand and fifty-four. (Prove by Division.)

CHAPTER VII.

PROVING BY THE NINES.

103. *What is the use of proving our work?*

A result in arithmetic is evidently useless unless correct; the labor devoted to it has been wasted, and the whole operation must be gone over again. In sending or paying a bill, for example, we must first be perfectly sure that it is made out right. In arithmetic the great requisite is strict accuracy.

104. *How do we insure accuracy in arithmetic?*

We generally succeed in obtaining a correct result by taking great care, by thoroughly understanding our work, and by an occasional re-examination. But as even the best is liable to make a mistake, we should always have the means of satisfying ourselves regarding the accuracy of our work.

105. *How do we insure accuracy in Addition and Subtraction?*

In Addition we can hardly be wrong when, upon adding each column twice, once up and once down, we obtain the same result each way. In Subtraction we are sure to be right when, on adding the Remainder to the Subtrahend, we have the Minuend. These modes of testing are very simple and cannot be shortened.

106. *How do we insure accuracy in Multiplication and Division?*

The surest way to test Multiplication is, as we have learned, by Division, and to test Division there is no way surer than by Multiplication. But in long operations these tests are too slow and troublesome. There is another mode of testing, much shorter and easier, and although not absolutely certain, exceedingly likely to be so. It is called the *Test by the Nines.*

107. *Why is it so called?*

Because a very curious property possessed by 9, and by no other figure, is employed in the operation. Take any number, 38, for example. Find the sum of its digits; $3 + 8 = 11$. To this number 38 add 9; the sum is 47. Find the sum of the digits; $4 + 7 = 11$, as before. To 47 add 9; the sum is 56; $5 + 6 = 11$. To 56 add 9; the sum is 65; the sum of the digits is still 11. To 65 add 9; the sum is 74; 11 is again the sum of the digits. And so on; no matter what the number is, adding 9 to it makes no change in the sum of its digits. If the unit is nought, of course the sum of the digits must be nine more than before; so that the proper way to state this peculiarity of the 9 is: *The addition of 9 to any number either makes the sum of the digits 9 greater or leaves it entirely unchanged.*

The reason of this peculiar property of the nine some young pupils may readily understand. Nine is equal to *ten* minus *one;* that is, to *one* ten minus *one* unit. In adding 9, therefore, to any number, I *add one* to the tens, and at the same time *take away one* from the units. I diminish the digits, therefore, just as much as I increase them, and consequently I make no change in their total number. For the same reason subtracting a nine from a number makes no change in the sum of its digits. In adding 9 I add *one* ten and subtract *one* unit; in subtracting 9 I subtract *one* ten and add *one* unit.

108. *What practical use do you make of this property of Nine?*

Since adding or subtracting 9 makes no change in the sum of the digits, I am certain that the digits of every multiple of 9 must amount to 9. For example: 18, $1 + 8 = 9$; 27, $2 + 7 = 9$; 36, $3 + 6 = 9$; 45, $4 + 5 = 9$; 54, $5 + 4 = 9$, etc. Consequently, when I count the digits of any number and find they amount to 9, or any multiple of 9, that number must be divisible by 9. For example: 144, 252, 369, 576, are certainly divisible by 9, because their digits amount either to 9, or 18 a multiple of 9.

Divisible by 9 simply means that 9 can be taken away a certain number of times, and that at last nothing will be left. If any number is left, the number is not divisible. For example: 27 is divisible by 9, because if 9 is taken away three times nothing is left. 42 is not so divisible, because if 9 is taken away four times, 6 is left.

109. *What other use do you make of this remarkable property of Nine?*

Since, by counting the digits, I can tell if a number is divisible by 9, evidently by the same test I can tell if a number is *not* divisible by 9. Take 684 for example. The digits added amount to 18, a multiple of 9. Therefore 9 can be taken away from 684 a certain number of times and nothing will be left. Now take 345. The digits added amount to 12, not a multiple of 9, there being 3 over. Now I am certain that if 9 is taken away a certain number of times from 345, there will at last be an Over of 3 left.

110. *Give another example.*

Take 528; the digits make 15; therefore there would be 15 left after taking away a certain number of Nines. Take another 9 away and 6 is left. Therefore 528 if divided by 9 will leave an Over of 6, as can be easily ascertained by actual trial. In the same way I find in a very short time that if 9 is divided into 865747 the Over will be exactly 1.

111. *How do you count these digits? Do you add them all up together and then throw out the Nines?*

It could be done that way, but it is shorter to cast the

Nines out as fast as we meet them. For example: $8 + 6 = 14$, rejecting the 9, 5; $5 + 5 = 10$, rejecting the 9, 1; $1 + 7 + 4 = 12$, rejecting the 9, 3; $3 + 7 = 10$, rejecting the 9, 1. Still shorter: $8 + 6 = 14 = 5$, $+ 5 = 10 = 1$, $+ 7 + 4 = 12 = 3$, $+ 7 = 10 = 1$.

In adding the digits, it shortens the work to skip the Nines or whatever figures amount to Nine. For example: Mere inspection is enough to show that the following large number is divisible by 9: 36547281279.

MODEL.

Find the Nine Over of 83596874312, and prove by actual Division.

$$8 + 3 = 11 = 2 + 5 + 6 = 13 = 4 + 8 = 12 = 3 + 7 = 10$$
$$= 1 + 4 + 3 + 1 = 9 = 0 + 2 = 2 \text{ Over.}$$

PROOF.

$$\begin{array}{r|l} 9 & \underline{83596874312} \\ & 9288541590 + 2 \text{ Over.} \\ & \underline{9} \\ & 83596874312 \end{array}$$

The same Over is found by rejecting the 9's as by actual division.

PRACTICAL QUESTIONS. No. 22.

COUNTING THE DIGITS TO FIND THE OVER OF NINE.

211. Find the Nine Over of 37489654, and prove by Division.

212. Find the Nine Over of 897236547, and prove by Division.

213. Find the Nine Over of 363872139, and prove by Division.

214. Find the Nine Over of 196354385, and prove by Division.

215. Find the Nine Over of 563287438, and prove by Division.

Divide 365728396 by 9, and prove by casting out the Nines.

MODEL.

$$\begin{array}{r|l} 9 & \underline{365728396} \\ & \;\;40636488 + 4 \text{ Over.} \\ & \underline{9} \\ & 365728396 \end{array}$$

PROOF.

$$3+6=9=0+5+7=12=3+2+8=13=4+3+6=13=4 \text{ Over.}$$

216. Divide 691792893 by 9, and prove by casting out the Nines.

217. Divide 786584382 by 9, and prove by casting out the Nines.

218. Divide 539836294 by 9, and prove by casting out the Nines.

219. Divide 117225333 by 9, and prove by casting out the Nines.

220. Divide 495693792 by 9, and prove by casting out the Nines.

112. *How do you make this property of Nine a test in Multiplication and Division?*

In a very simple manner, as a short example will show. Suppose that, having multiplied 1489 by 1283, and finding the product to be 1910387, I wish to test the correctness of this product without the trouble of division. I cast the Nines out of the Multiplicand; 4 is over. I cast the Nines out of the Multiplier; 5 is over. I multiply these Overs; 20 is the result, or 2, with the Nines cast out. Now 2 must be over in the Product if I am right. I cast the Nines out of the Product, and find 2 to be the Over. Consequently I conclude that in all probability I am right.

113. *In all probability! Why don't you say absolutely right?*

Because if a Nine accidentally slipped into the work or slipped out of it, the 2 would still be over, though the result was wrong. But such a singular mistake is so unlikely to occur that the Nine Test may generally be taken as a very satisfactory proof of the correctness of our work.

114. *Give a practical example of testing by the Nines.*

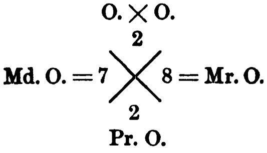
Make an oblique cross to the right of the work. In its left angle set the Over of the Multiplicand; in its right angle set the Over of the Multiplier. In the upper angle set the Over of the product of these two Overs, and in the lower angle set the Over of the total Product. If the lower angle number is the same as that in the upper angle, the work is most probably right.

MODEL.

Multiply 678427 by 58643, and prove by the Nines.

```
     678427 Multiplicand Over = 7
      58643 Multiplier Over = 8
    -------
    2035281
   2713708
  4070562
 5427416
3392135
-----------
39784994561 Product Over = 2
```

PROOF.



The same Overs are found in the upper and lower angles.

EXPLANATION.—Set the Md. Over 7 in the left angle of the cross; set the Mr. Over 8 in the right angle. Multiply these Overs, and set in the upper angle 2, the Over of their product $56 = 5 + 6 = 11 = 2$. Find the Over of the Product; it is 2. Set this 2 in the lower angle. It agrees with the number in the upper angle; therefore the work is right almost beyond a doubt.

PRACTICAL QUESTIONS. No. 23.

PROVING MULTIPLICATION BY THE NINES.

221. Multiply 71238 by 56644, and prove by the Nines.
222. Multiply 44397 by 47523, and prove by the Nines.
223. Multiply 68228 by 71286, and prove by the Nines.
224. Multiply 5934006 by 32478, and prove by the Nines.
225. Multiply 8796543 by 6834569, and prove by the Nines.
226. Multiply 397283 by 548372, and prove by the Nines.
227. Multiply 667789 by 825796, and prove by the Nines.
228. Multiply 834567 by 3695840, and prove by the Nines.
229. Multiply 365789 by 743958, and prove by the Nines.
230. The Multiplicand is the nine digits in order; the Multiplier is the nine digits reversed; find the Product, and prove by the Nines.

115. *Can the correctness of a Division question be also tested by the Nines?*

It can, and in almost the same way; but we must remember that the Dividend corresponds with the Product, and the Divisor and Quotient with the Factors. Consequently we set the Over of the Dividend in the lower angle; then, setting the Over of the Divisor to the left and that of the Quotient to the right, we multiply them, and set their Over in the upper angle. If the upper and lower angle agree, we may presume that the work is right.

MODEL.

Divide 148276 by 38, and prove by the Nines.

2 = Over of 38 | 148276 | Dividend Over = 1
114 | 3902 Quot. Over = 5
342
342
76
76

PROOF.

1 = O. × O.
Dr. O. = 2 ╳ 5 = Quo. O.
1
Dd. O.

Always set in the right and left angles the Factors whose Overs are to be multiplied. These are generally different, but those in the upper and lower angles must be alike if the work is right.

116. *Suppose there is a remainder after the division; what is to be done with it?*

The Remainder being an Over, its Over is to be added to the Over of the Divisor and Quotient; the sum is then set in the upper angle and compared with that in the lower.

MODEL.

Divide 256 into 2137965, and prove by the Nines.

4 = Over of 256 | 2137965 | Dividend Over = 6
2048 | 8351 Quot. Over = 8
899
768
1316
1280
365
256
109 = 1 Over.

PROOF.

O. × O. = 5 + 1 = O. O.
Dr. O. = 4 ╳ 8 = Quo. O.
6
Dd. O.

Over in lower angle 6 = 5 + 1 Overs as in upper angle.

EXPLANATION.—Over of Dividend 6 is set in lower angle; Over of Divisor 4 is set in left angle, and Over of Quotient 8 is set in right angle. Multiply the two last Overs, and set their Over 5 in upper angle. To this Over add the Remainder Over 1. 5 + 1 in the upper angle corresponds with 6 in the lower, and the work may be considered right.

Before attempting work of his own, the Pupil should be able to readily explain all the above.

PRACTICAL QUESTIONS. No. 24.

PROVING DIVISION BY THE NINES.

231. Dividend = 3085326; Divisor = 5682. Quotient = ? Prove by the Nines.

232. Dividend = 216912714; Divisor = 71873. Quotient = ? Prove by the Nines.

233. Dividend = 14963776; Divisor=7328. Quotient = ? Prove by the Nines.

234. Dividend = 417219; Divisor = 6014. Quotient = ? Prove by the Nines.

Remember what is to be done with Remainder Over.

235. 695632 ÷ 7946 = ? Prove by the Nines.
236. 372847 ÷ 1239 = ? Prove by the Nines.
237. 1726542 ÷ 9847 = ? Prove by the Nines.
238. 1019486 ÷ 92786 = ? Prove by the Nines.
239. 22445664 ÷ 21984 = ? Prove by the Nines.
240. 17960387 ÷ 13086 = ? Prove by the Nines.

PRACTICAL QUESTIONS. No. 25.

REQUIRING FAMILIARITY WITH THE FOUR RULES AND SOME JUDGMENT IN APPLYING THEM.

Quotient = 186; Divisor = 1607; Remainder = 47; what is the Dividend? (Prove.)

HINT.—The young student is sometimes puzzled by questions containing high numbers, though he could easily manage them if the numbers were small. He must remember that *questions of the same kind are always solved in the same way,* whether the numbers are high or low. While doubtful what to do, therefore, he should ask himself a question of *exactly the same kind*, but with small figures. For example: Before undertaking the above, he should ask himself such a question as this: The Quotient is 5; the Divisor is 4; the Remainder is 3; what is the Dividend? Setting it down on his slate as follows may make it still clearer: $\begin{array}{l|l} 4 & \phantom{5+3\text{ over.}} \\ \hline & 5+3\text{ over.}\end{array}$ That is, what is the number into which 4 will go five times and leave 3 over? The number is evidently 23. How is it obtained? By multiplying the Divisor and Quotient together, and adding the Remainder to the Product: $4 \times 5 = 20$; $20 + 3 = 23$. Question 241 should be done in a precisely similar manner.

$$\begin{array}{rrl}
 & 1607 & = \text{Divisor.}\\
 & \underline{186} & = \text{Quotient.}\\
 & 9642 & \\
 & 12856 & \\
 & \underline{1607} & \\
 & 298902 & = \text{Product.}\\
 & \underline{47} & = \text{Remainder.}\\
\textit{Ans.} & 298949 & = \text{Dividend or number sought.}
\end{array}$$

PROOF.

$$\begin{array}{r|r|l}
\text{Divisor} = \underline{1607} & 298949 & = \text{Dividend.}\\ \cline{3-3}
 & \underline{1607} & \underline{186 = \text{Quotient.}}\\
 & 13824 & \\
 & \underline{12856} & \\
 & 9689 & \\
 & \underline{9642} & \\
 & 47 & = \text{Remainder.}
\end{array}$$

241. Divisor = 6246; Quotient = 472; Remainder = 1709; Dividend = ? (Prove.)

242. Dividend = 16 millions; Quotient = 1625; Divisor = ? Remainder = ? (Prove.)

243. The Quotient is double the Divisor, and three times the Remainder; what is the Dividend if the Remainder is 256? (Prove.)

244. The Divisor is seven times the Quotient; the Quotient

is ten times the Remainder; the Remainder is 2345; what is the Dividend? (Prove.)

245. Divide the product of the first six digits by their sum. Prove by the Nines.

246. What number multiplied by 64 produces a million? (Prove.)

247. The Product of two Factors is 1520694; half of one of the Factors is 3129; what is the other Factor? (Prove.)

248. 50738 × 40062 = ? Prove by Division.

This exercise is given to make the Pupil careful regarding the Noughts.

249. 61875 is to be multiplied by 287248; the Product is to be divided by 12375. Quotient required. Prove Multiplication and Division by the Nines.

250. There are two numbers: 825 and 9318375. The sum of their sum, of their difference, of their product, and of their quotient, required. *Ans.* 7706307420.

This need not be proved. The Pupil able to do it is now quite ready for some practical application of the FOUR RULES.

CHAPTER VIII.

PROBLEMS.

(To be solved by means of the Four Rules.)

117. *What is a Problem?*

In school language a Problem is a piece of work to be done by the Pupil, in order to learn how to make proper use of certain rules in which he has received instruction. A Problem is solved by thinking, that is, by using our minds.

A Problem is work to be done: the rules are the tools by which we do the work: but the mind is the power that uses the tools, and without the exertion of which all rules are useless.

118. Was not Question 250 a Problem?

No; there you were told what numbers were to be added, subtracted, multiplied, or divided, and, of course, you had nothing more to do than to apply the rules. But in a problem you have to find for yourself *what* is to be added, subtracted, multiplied, or divided—which can be done by thinking only, as was said before, and for thinking, no rules can be given but care and plenty of practice.

119. Give an example of a Problem.

The following question is one: A farmer bought 8 horses at 95 dollars each. In paying for them he found that he had only 692 dollars in cash, but he had plenty of corn worth 2 dollars a bushel. The seller being willing to accept corn for the balance, that is the rest of the debt, the question is, how many bushels was he entitled to receive?

The Pupil should be able to state this question readily, that is, he should learn it by heart. If he is not able to say it by heart, he does not understand it, and of course he cannot perform it. The mere effort made by our memory in order to recite correctly is a great stimulus to thought, and wonderfully aids us in the solution of a difficulty.

Few Pupils are stupid at Arithmetic, but many are lazy; that is, they prefer guessing to giving themselves the trouble of thinking.

A Pupil therefore should never ask the Teacher's assistance until he is able to state the question aloud, clearly and readily, without the aid of the book.

EXPLANATION.—In solving the above question, a little thinking tells us that the price of 8 horses, at 95 dollars each, must be 95 × 8 or 760 dollars—the total debt. Of this debt 692 dollars being paid in cash, the balance due must be 760 — 692, or 68 dollars, to be paid in corn. Corn being 2 dollars a bushel, the number of bushels to be given depends evidently on the number of times that 2 dollars are contained in 68 dollars. If 10 dollars were the balance, 5 bushels would pay it; if 24 dollars were the balance, it would be paid by 12 bushels. We have, therefore, to find how many times 2 dollars are contained in 68 dollars—an operation to be performed by Division. 68 divided by 2 equals 34; therefore 34 bushels, at 2 dollars each, will discharge the debt.

The above is the reasoning, but the actual work may be shown as in the following

MODEL.

$95	$760	2 | 68
8	$692	34, or 34 bushels.
$760 = total debt.	$68 = balance due.	

Answer. 34 bushels at $2 each.

The sign $ stands for dollars. It is an abbreviation of 8 r. (meaning eight reals), the mark by which the Spaniards, the great Atlantic traders of three hundred years ago, denoted their favorite silver coin, called a *Piece of Eight.*

The Spanish *real* is a small silver coin, still worth about 12½ cents.

PRACTICAL QUESTIONS. No. 26.

PROBLEMS REQUIRING A KNOWLEDGE OF THE FOUR RULES.

251. In a school of a thousand pupils, 27 were over 18 years of age, 156 over 17, and the rest were under 16. How many pupils were under 16 years of age? (Prove.)

252. A merchant spent $246 in buying coffee at 25 cents a pound. How many pounds should he receive for his money? (Prove.)

As will be soon explained, $246 equals 24600 cents.

253. A grocer spent $320, or 32000 cents, in buying sugar at 5 cents a pound. He sold 1250 pounds at 6 cents a pound, 1350 pounds at 7 cents a pound, and the rest at 8 cents a pound. What was his total profit? (Prove.)

254. A merchant bought 47 casks of wine at 98 dollars per cask. 20 proving leaky, he was compelled to sell them at 88 dollars per cask. The remainder he sold at 128 dollars per cask. What was his profit? (Prove.)

255. A shirt-maker pays 600 dollars for 6000 yards of muslin, and 500 dollars for 1000 yards of Irish linen; he also pays a dollar for the making, washing, and ironing of each shirt. Supposing 3 yards of muslin and a half yard of linen to be required for each shirt, and that he sells them all within a month at 2 dollars apiece, what profit has he made by the venture, allowing 50 dollars for the monthly rent of the store, and 50 dollars for the clerk's salary? (Prove.)

256. A young clerk, who earned 600 dollars a year, paid 6 dollars a week for his board, 3 dollars a month for his washing, 60 dollars a year for his clothing, 1 dollar a week for books and papers, 1 dollar a week for his spending money, and 10 dollars a year for charity. As he neither drank liquors nor used tobacco, he was also able to deposit every week 1 dollar in the saving fund. Whatever money he had left he spent in buying Christmas presents for his father and mother, who lived in a little village twenty or thirty miles from the city. How much money did he have left to devote to the Christmas presents? (Prove.)

A year contains 52 weeks.

257. A gentleman, owing some bills in the city, ordered his son to pay them, giving him a 100 dollar note for the purpose. In the book-store the boy paid $3.86; in the drug-store, $8.42; in the grocer's, $25.47; in the dry-goods store, $19.48; and at the coal merchant's he was to order and pay for 2 tons of coal at $6.50 per ton. He was also ordered to buy 56 postage stamps at 2 cents each, and 56 at 1 cent each. He was allowed 12 cents for car fare, but, being strong, healthy, and fond of walking, he did not ride in the cars, and his father would not take back the 12 cents. How much change should he return to his father? (Prove.)

Remember that 100 dollars = 10000 cents.

258. A small dealer spends 2 dollars on pictures at 5 cents apiece; 3 dollars on tops at 12 cents apiece; and 4 dollars on

books at 16 cents apiece. How many pictures, tops, and books does he buy? (Prove.)

259. Every common year has 365 days. How many days is a boy 12 years of age older than a boy 9 years of age?

260. A boy, who lives 1 mile from his school, walks there every day and also walks back. His schooltime is 10 months in the year, each month having 4 weeks, and each week having 5 days. In how many years will this boy have walked 2000 miles on his way to and from school? (Prove.)

120. *Some boys are very quick and correct in doing long sums on the slate, but, when asked by their father to solve some simple question, without slate or pencil, they are so puzzled that they do not know what to say. Why do they all at once appear to be so dull?*

They appear to be dull because they have been relying too much on their pencil and too little on their head. They are not at all dull; but, if the Father thinks that they have been badly taught, he is not always in the wrong. The Teacher, finding such boys usually very quick and industrious, and thinking *head questions* too easy for them, seldom or never gives them any questions but such as require the help of slate and pencil for solution. The Father, not knowing the real state of the case, is naturally disappointed, and just as naturally finds fault with the Teacher.

121. *What should a Pupil do who wishes to satisfy his Father's reasonable expectations, and so be a credit to his Teacher?*

He should often practise himself in head work, or Mental Arithmetic, as it is called; that is, he should often try to solve little problems difficult enough to require careful thinking, but easy enough to be done without slate or pencil.

122. *Give one such problem, by way of example.*

A man buys 5 newspapers at 3 cents each, an account-book at 35 cents, writing-paper for 15 cents, a penholder, pens, and ink for 20 cents, and gives a dollar note in payment. What change should be returned?

Solution.—5 newspapers at 3 cents each cost 15 cents; 15 + 35 = 50; 50 + 15 = 65; 65 + 20 = 85 cents, the money to be paid. 100 cents — 85 cents = 15 cents = the change to be returned.

Proof.—85 cents + 15 cents = 100 cents = 1 dollar.

PRACTICAL QUESTIONS. No. 27.

PROBLEMS TO BE SOLVED WITHOUT SLATE OR PENCIL.

261. A man bought 6 quires of paper for 48 cents, and sold them so as to gain 18 cents. How much did he charge for each quire? (Prove.)

262. A dealer bought a piano at auction for 150 dollars, and paid 5 dollars for having it removed to his store. He hired it to a music pupil at the rate of 10 dollars a month, and again paid 5 dollars for the removal. At the end of six months the pupil was so well pleased with the piano that he bought it for 180 dollars. What was the dealer's profit? (Prove.)

263. Two men start at 6 o'clock in the morning; one travels eastwardly at 3 miles an hour, and the other westwardly at 4 miles an hour. At 1 o'clock in the afternoon they stop for dinner. How far are they apart? (Prove.)

264. A father gives his son a 2-dollar bill to buy 3-cent and 2-cent postage stamps, of each an equal number. How many of each kind are to be brought back? (Prove.)

265. A farmer had 48 eggs, worth 2 cents apiece; these he exchanged with a grocer for coffee worth 31 cents a pound. How much coffee and change was he to receive? (Prove.)

266. A father and his two sons are employed in a printing-office, where the father is paid three times as much as the two sons together. The sons are paid 60 cents a day each. What are the family's earnings every week? (Prove.)

267. A man bought two horses, paying 75 dollars for each. He sold one for 100 dollars, and the other for 72 dollars. What was his profit? (Prove.)

268. Two boys start at 9 o'clock in the morning, one going north at 2 miles an hour, the other going south at 3 miles an hour. Supposing that they rest no more than one hour for dinner, at what o'clock will they be 40 miles apart? (Prove.)

269. A dealer picked up 10 pictures at 6 dollars apiece. One he sold for 20 dollars, and another for 18 dollars; for the rest he could get no more than 5 dollars each. What was his profit? (Prove.)

270. A boy, living 15 miles from the city, travels by rail to school and back five days every week. The regular fare is 3 cents a mile, but, buying school tickets, he can ride for 2 cents a mile. How much does he save in 40 weeks by buying school tickets? (Prove.)

An endless number of variations may be made on the above, none of course difficult enough to require slate and pencil, and all to be done aloud in class. Many pupils require careful drilling in simple problems, and the simpler these are the better they effect their purpose—that of teaching the Pupil to think for himself.

In performing them, a Pupil should never be allowed to count on his fingers; doing so spoils everything by making him imagine he can calculate without thinking.

PLUS WORK.

123. *What is meant by Plus Work?*

We have had already examples of Plus Work in Addition and Subtraction on page 27. There, however, we were limited to the use of the Plus and the Minus signs. Plus Work generally is intended to give us practice in the use of all the signs used in Arithmetic.

124. *Give an example of the use of Plus Work.*

$(\frac{45}{9} - 3) \times 8 = 10 + 6$. This is a very short way of expressing that 8 times the remainder, left after taking 3 from the quotient of 45 divided by 9, is equal to the sum of 10 and 6. Speaking directly to us through our eyes, it is also evidently a much quicker way of expressing the fact than doing so by words, which are comparatively cumbersome, and therefore make their way much more slowly to our understandings. But to accustom ourselves to the ready use of signs, we must have some practice in Plus Work.

125. *Repeat the signs the use of which we have so far learned.*

They are six, namely: Plus (+), the sign of Addition; Minus (—), the sign of Subtraction; Multiplied by (×), the sign of Multiplication; Divided by (÷), the sign of Division, Equal (=), the sign of equality; and Question (?), asking, or How many?

The Pupil will remember that another way of expressing Division is to place the Dividend above the bar, and the Divisor below it: as $\frac{48}{12}$, which means that 48 is to be divided by 12.

126. *Now mention the signs of which we have not yet learned the use.*

Few others seem necessary, still there is one ( ) whose use must be well understood. It is called Vinculum or Brackets, and it serves to keep together the quantities on which the signs operate.

127. *Give an example of Vinculum or Brackets.*

Examine the following expressions: $(3 + 4) \times 9 = 63$, and $3 + 4 \times 9 = 39$. One means that the sum of 3 and 4 is to be multiplied by 9. The other means that 3 is to be added to the product of 4 times 9. This difference is shown by placing 3 + 4 within the Vinculum.

128. *What, then, is the use of the Vinculum or Brackets?*

Without the Vinculum, the Multiplication sign or the Division sign affects only the two quantities which it unites. With the Vinculum, these signs affect all the numbers included in the Vinculum. Example: $7 + 9 \times 5 = 52$. This is right, because the 5 is multiplied by 9 only. $(7 + 9) \times 5 = 80$. This is right, because 5 is to be multiplied by the sum of 7 and 9.

Generally, Vinculums are needless unless with the signs of Multiplication or Division.

Another form of the Vinculum is a line or bar drawn over the numbers to be connected, as: $\overline{5 + 9} \div 2 = 7$. The chief use of a Bar, however, is when a Vinculum lies within a Vinculum, as will afterwards be explained.

129. *Give a Model for Plus Work.*

MODEL.

Question: $(246 + 75) \times 7 + 12 - 16 \div 2 = ?$

To be read orally (not written) as follows: *To the product* of the sum of 246 and 75 multiplied by 7, add 12, and from this sum subtract 16 divided by 2.

$$\begin{array}{rll} 246 & \qquad 16 \div 2 = 8 & \qquad 12 - 8 = 4 \\ \underline{75} & & \\ 321 & & \\ \underline{7} & & \\ 2247 & & \\ \underline{4} & & \\ 2251 = \textit{Ans.} & & \end{array}$$

Explanation.—Finding the sum of 246 and 75 to be 321, I multiply it by 7, obtaining 2247. Before adding 12, I see that it is to be diminished by the half of 16, or 8. This leaving only 4 to be added, the final result is 2251.

PRACTICAL QUESTIONS. No. 28.

PLUS WORK, TO BE READ, DONE ON THE SLATE, AND EXPLAINED.

271. $238 + 179 - 18 \times 9 = ?$ (Read.) *Ans.* 255.

272. $(648 \times 240) + (216 + 80) \times 9 = ?$ (Read.) *Ans.* 311040.

273. $648 \times (240 + 216) + 80 \times 9 = ?$ (Read.) *Ans.* 296208.

274. $\overline{648 \times 1512} \div \overline{9 \times 18} = ?$ (Read.) *Ans.* 20244.

275. $3 + 5 + 8 \times 9 + 6 + 14 - \overline{2 \times 8} = ?$ (Read.) *Ans.* 84.

276. $3 + (5 + 8) \times 9 + (6 + 14 - 2) \times 8 = ?$ (Read.) *Ans.* 264.

277. $(3 + 5 + 8) \times 9 + (6 + 14) - 2 \times 8 = ?$ (Read.) *Ans.* 148.

278. $3 + 5 + 8 \times (9 + 6) + (14 - 2) \times 8 = ?$ (Read.) *Ans.* 224.

279. $(8 + 12) \div 4 + (27 - 3) \div 8 - \frac{1024}{128} + \frac{936}{117} = ?$ (Read.) *Ans.* 8.

280. $(37 - 19) \div 9 \times \frac{650}{87-74} = ?$ (Read.) *Ans.* 100.

The few other signs used in Arithmetic will be explained as they occur.

CHAPTER IX.

ANALYSIS.

130. What is Analysis?

Analysis in Arithmetic teaches us how to discover what is unknown by means of what is known. Suppose we are asked the following question: If 7 men earn 28 dollars by digging a long trench, to how many dollars are 3 of these men entitled? Here what the 7 earn is *known;* what the 3 earn is *unknown,* and that is what we are required to find. Now, instead of trying to solve the question at one jump by attempting to grasp the relation between 7 and 3, which at present would be too difficult for us, we try to overcome the difficulty by doing as a little boy did on coming to a stream that, though shallow, was too wide for him to jump. Throwing a big stone into the middle of the water, he stepped first on the stone, and thence easily to the opposite bank. In the same way we divide the hard question into two easy questions. We first ask, if 7 men earn 28 dollars, what does ONE man earn? Of course, the seventh part of 28, that is, 28 ÷ 7, or 4 dollars. That is step first. Now for the second question: If one man is entitled to 4 dollars, to how much are 3 men entitled? Evidently to three times as much, that is, to 4 × 3, or 12 dollars, the answer.

Solving the question in this way, by making two offers at it, is called *Analysis*, meaning division, because it divides the greater difficulty into two smaller difficulties, which the mind can readily overcome.

131. *Show and explain how such mental work is done on the slate.*

Such work is represented on the slate as follows:

Men.	$	
7	28	28 ÷ 7 = 4
1	4	4 × 3 = 12
3	12	*Ans.* $12.

EXPLANATION.—Here are two columns, one for the men, the other for the dollars. In what order they are to be placed depends on the nature of the question. *The column on the right is always to contain the unknown quantity.* In the present question, the unknown quantity being money, the three men's pay, the dollar column goes to the right. So we set 7 men at the head of one, and 28 dollars at the head of the other. Then we ask ourselves, if 7 men earn 28 dollars, what will ONE man earn? Evidently seven times less than 28; so we set 1 in one column, and 4 in the other. Lastly, we ask, if ONE man earns 4 dollars, how many will 3 men earn? Clearly 12 dollars; so, setting 3 in one column, and 12 in the other, the work is done.

132. *Give another example in Analysis.*

Suppose the following to be the question: If 8 pounds of coffee cost 160 cents, what is the price of 17 pounds? First, money being required, the cent column goes to the right, and the pound column to the left; so I set 8 in one column, and 160 in the other. 8 pounds costing 160 cents, ONE pound must cost 160 ÷ 8, or 20 cents; 1, therefore, is set in the left, and 20 in the right hand column. ONE pound costing 20 cents, 17 pounds must cost 20 × 17, or 340 cents. 17, therefore, being set in the pound column, and 340 in the cent column, the unknown quantity is found, and the work is done.

WORK.

Pounds.	Cents.	
8	160	$160 \div 8 = 20$
1	20	$20 \times 17 = 340$
17	340	*Ans.* 340 cents.

133. *How is this proved to be the right answer?*

By reversing the operation as follows:

PROOF.

Pounds.	Cents.	
17	340	$340 \div 17 = 20$
1	20	$20 \times 8 = 160$
8	160	

EXPLANATION.—If 17 pounds cost 340 cents, ONE pound must cost 17 times less, or 20 cents, and 8 pounds will cost 8 times as much, or 160 cents.

134. *I notice that in your examples so far given the first difficulty is overcome by Division, and the second by Multiplication: is this the case in all Analysis questions?*

In most questions it is, but in some it is exactly the opposite. For example: A citizen has paid a farmer a certain sum of money for the privilege of keeping 8 horses at pasture for 30 days. How long has he the right to keep 12 horses at pasture on the farm? Here days being required, 30 is set to the right and 8 to the left. I then ask, if 8 horses may remain for 30 days, how long may ONE horse alone remain?

He has evidently the right to remain 8 times longer, that is, 8 times 30, or 240 days. Therefore, 1 is set in the left and 240 in the right. If ONE may remain 240 days, how long may 12 remain? Evidently for a time 12 times shorter, or 240 ÷ 12, that is, 20 days.

WORK.

Horses.	Days.	
8	30	30 × 8 = 240
1	240	240 ÷ 12 = 20
12	20	

Ans. 12 horses may pasture 20 days.

PROOF.

Horses.	Days.	
12	20	12 × 20 = 240
1	240	240 ÷ 8 = 30
8	30	

Here, contrary to the previous questions, the first answer is obtained by Multiplication, and the second by Division. This shows that Analysis is not a mechanical rule, and that, therefore, the Pupil must carefully study the nature of the question.

135. *So far I think this is all clear enough. But suppose the Divisor was too great for the Dividend, or that it would not go evenly: in one case no division would be possible, in the other there would be a puzzling Over. In either case what should be done?*

In either case a difficulty would certainly remain too great for your present knowledge of the Four Rules to overcome, and which, therefore, must be deferred until you learn FRACTIONS.

Of course, all extra difficulties will be avoided in the following Analysis questions, since we learn best when we learn one thing at a time.

PRACTICAL QUESTIONS. No. 29.

IN ANALYSIS.

When entering the following in his copy-book, the Pupil carefully imitates the last Models, and passes no difficulty without giving a satisfactory explanation to the Teacher.

281. A locomotive can travel 288 miles in 18 hours; how many miles can it travel in 30 hours? (Prove.)

282. A locomotive can travel 456 miles in 19 hours; how many miles can it travel in 48 hours? (Prove.)

283. If 15 workmen require 32 days to do a piece of work, in how many days would 16 men be able to finish it? (Prove.)

284. If 15 men require 32 days to do a piece of work, how many men would be able to finish it in 96 days? (Prove.)

285. A black horse runs 64 miles in 4 hours; a brown horse runs 144 miles in 18 hours. How much faster does one horse travel than the other? (Prove.)

The above question differs a little from the others, but a thoughtful Pupil will not find it difficult.

286. If 33 tons of merchandise cost 41448 cents, what will 784 tons cost? (Prove.)

287. If 75 tons of merchandise cost 18300 cents, how many tons may be bought for 84180 cents? (Prove.)

288. If 128 yards of muslin are required for 32 shirts, how many shirts may be made out of a million yards? (Prove.)

289. If 6 cats can kill 18 rats in one day, how many rats can 8 cats kill in two days? (Prove.) *Ans.* 48 rats.

290. If 6 cats can kill 30 rats in one day, how many rats can 10 cats kill in 7 days? (Prove.) *Ans.* 350 rats.

The Pupil has now learned the principles of the Four Rules, and has also had some practice in applying them. Still this First Part of our Arithmetic would not be complete without giving him a correct idea of how BILLS should be made out, especially as for many kinds of Bills no knowledge whatever of Fractions is necessary.

CHAPTER X.

BILLS.

136. *In order to be able to make out a Bill properly, what must I first understand?*

You must understand the different denominations of the United States Money, of which the following is the

TABLE.

10 Mills make	1 Cent.
10 Cents make	1 Dime.
10 Dimes make	1 Dollar.
10 Dollars make	1 Eagle.
Also 100 Cents make . . .	1 Dollar.

QUESTIONS on the above Table: How many Mills in 2 Cents? in 4 Cents? in 7 Cents? How many Cents in 3 Dimes? in 5 Dimes? in 9 Dimes? in 1 Dollar? in 4 Dollars? in 8 Dollars? in a Quarter Dollar? in 2 Quarter Dollars? in 3 Quarter Dollars? How many Dollars in 2 Eagles? in 5 Eagles? in 7 Eagles? in 9 Eagles? How many Dimes in 2 Dollars? in 5 Dollars? in 8 Dollars? in 1 Eagle? in 3 Eagles? in 7 Eagles?

Similar questions to the above may be given until the Pupil shows by ready and correct answering that he thoroughly understands the Table.

137. *Is United States Money easy to understand?*

Yes; it is easy to understand and particularly easy to note down, as its different denominations follow the Arabic system of increasing or decreasing in value by tens. Therefore it is enough to know the denomination of one figure alone. That of the others can then be known from their position.

138. *How is the value of one figure denoted?*

Chiefly in two ways: first, by setting the dollar mark ($) before the number, as $5678, read 5678 dollars. Here there is no difficulty. Second, by setting after the number the denomination of its last figure, as 5678 cents. Here, however, there is a difficulty.

139. *What is the difficulty?*

If we read it 5678 cents, we are not clear. If we read it 5 eagles, 6 dollars, 7 dimes, and 8 cents, we are slow. People, therefore, have rejected both ways. In fact, of the five denominations of U. S. Money, *two* only are in general use. Eagles, dimes, and mills are spoken of comparatively seldom. Generally, the two denominations of Dollars and Cents are found sufficient for all purposes.

140. *How is this managed so as to avoid mistakes?*

Mistakes are avoided by the careful use of a mark called the *dollar dot.* Since 100 cents make a dollar, we turn cents into dollars by dividing by 100. For example: 300 cents make 3 dollars; 500 cents make 5 dollars, and so on. But in dividing by 100 we know already that the quotient is the number left after cutting off two figures on the right. Therefore, in order to turn cents into dollars, no more need be done than cut off the two figures on the right by the dollar dot. These two figures will then denote the cents; whatever is left shows the dollars. For example: 378 cents = $3.78, or 3 dollars and 78 cents. 4567 cents = $45.67, or 45 dollars and 67 cents.

The Pupil must remember that, when the denomination is cents, *two* figures must be cut off to the right by the dollar dot.

This information regarding U. S. Money must be sufficient for the present, the subject being fully treated of hereafter in PART THIRD, Denominate Numbers.

141. *Give a practical example in U. S. Money.*

Suppose we were asked the amount of the following bill:

37 pounds of coffee at 24 cents a pound; 168 pounds of sugar at 9 cents a pound; and 75 pounds of tea at 56 cents a pound. The work would be done as follows:

$$\begin{array}{rcl}
37 \times 24 = 888 \text{ cents} &=& \$8.88 \\
168 \times 9 = 1512 \quad \text{“} &=& \$15.12 \\
75 \times 56 = 4200 \quad \text{“} &=& \underline{\$42.00} \\
 & & \$66.00 = \textit{Ans.}
\end{array}$$

In the same way find the amount of the following items: 1784 articles at 15 cents each; 9632 articles at 24 cents each; 1286 articles at 37 cents each; and 1593 articles at 49 cents each. *Ans.* $3835.67.

142. ***Now explain what is meant by a Bill.***

Suppose that on the 7th of April, 1883, a storekeeper, named William Jamison, sold to Francis Sawyer the following goods: 50 pounds of sugar at 9 cents a pound; 18 pounds of butter at 35 cents a pound; 48 yards of cotton drill at 7 cents a yard; 15 yards of cloth at $3 a yard; and 7 dozen of handkerchiefs at 96 cents a dozen. Mr. Jamison would write out for Mr. Sawyer a Bill of the above sale in the following manner:

PHILADELPHIA, *April 7, 1883.*

Mr. Francis Sawyer,

Bought of WILLIAM JAMISON,
217 FERNWOOD STREET.

50 pounds of sugar,	*at*	*9 cents,*	*$4*	*50*
18 pounds of butter,	*at*	*35 cents,*	*6*	*30*
48 yards of cotton drill,	*at*	*7 cents,*	*3*	*36*
15 yards of cloth,	*at*	*$3.00,*	*45*	*00*
7 dozen of handkerchiefs,	*at*	*96 cents,*	*6*	*72*
			$65	*88*

The above is the usual form of a Bill. It shows in a clear manner to Mr. Sawyer what he has bought and what he owes.

When Mr. Sawyer pays the Bill, Mr. Jamison writes "Received payment," and signs the Bill with his name, *William Jamison,*

written in the right hand lower corner. A Bill thus signed is called a RECEIPT.

Before entering the following Bills in his copy-book, the Pupil should make them out neatly on his slate, so that they may be correctly copied off.

PRACTICAL QUESTIONS. No. 30.

BILLS AND RECEIPTS.

291. Robert Furniss bought of Lewis Wallace, 1610 William Street, New York, January 10, 1884, the following bill of goods: 164 articles at 12 cents each; 1728 articles at 25 cents each; 1134 articles at 11 cents each; 724 articles at 7 cents each; and 1652 articles at 18 cents each. Make out a Bill, so that the Buyer may know what he owes the Seller. *Ans.* $924.46.

292. Philip Mangan purchased, February 10, 1884, the following from Albert Sullivan, 26 York Street, Philadelphia: 513 articles at 174 cents each; 137 articles at 405 cents each; 106 articles at 18 cents each; 1479 articles at 39 cents each; and 1656 articles at 5 cents each. Make out the Bill, and receipt it, Mangan having paid it at once. *Ans.* $2126.16.

293. Of John Purcell, 27 Grove Street, Chicago, Paul Sexton bought on March 17, 1884: 12 First Readers at 27 cents each; 18 Second Readers at 35 cents each; 29 Third Readers at 45 cents each; 36 Fourth Readers at 72 cents each; and 47 Fifth Readers at 108 cents each. Make out Bill. *Ans.* $488.07.

294. The Continental Hotel bought of James Watson, 1296 Wallace Street, Philadelphia, on May 12, 1880, the following: 144 pounds of beef at 18 cents per pound; 315 pounds of veal at 20 cents per pound; 3 dozen kidneys at 105 cents per dozen; and 16 sweetbreads at 30 cents each. Make out Bill. *Ans.* $

295. March 1, 1883, Mrs. Baker, 1397 Cedar Street, Boston, wished to receive payment from Mr. Ford, one of her board-

ers, for the following items: Board for 8 weeks at 550 cents per week; gas for 4 weeks at 30 cents per week; fire for 5 weeks at 80 cents per week; and washing 9 dozen of articles at 90 cents per dozen. Make out the Bill in a shape proper to be presented to Mr. Ford. *Ans.* $

296. William Bright bought of McGlensey & Son, 16 Front Street, Philadelphia, various goods as follows: 75 pounds of sugar at 9 cents a pound; 84 pounds of coffee at 27 cents a pound; 64 pounds of tea at 45 cents a pound; 72 pounds of cheese at 11 cents a pound; and 105 pounds of crackers at 18 cents a pound. Make out Bill and Receipt. *Ans.* $

297. April 5, 1880, Messrs. P. Cunningham & Son, 175 Market Street, Wilmington, Delaware, sold as follows to John Huneker: 5 dozen gloves at 210 cents a dozen; 15 dozen straw hats at 275 cents per dozen; 18 dozen stockings at 162 cents per dozen; 25 dozen linen collars at 144 cents per dozen; and 45 dozen rubber combs at 364 cents per dozen. Make out Bill. *Ans.* $

298. April 25, 1876, F. X. Brennan, 213 Platt Street, Baltimore, sold to George A. Batterbury: 45 barrels fish at $6.50 per barrel; 86 barrels flour at $5.75 per barrel; 64 barrels pork at $14.25 per barrel; and 86 barrels cornmeal at $4.25 per barrel. Mr. Batterbury pays $1000 on account. Make out Bill, showing how much he still owes. *Ans.* $

299. Frank Ward, coal merchant, 14 Peck Slip, New York, sold, November 2, 1881, as follows to Lorenzo Reich: 37 tons egg coal at $6.50 per ton; 72 tons stove coal at $5.95 per ton; 16 tons nut coal at $5.15 per ton; and 17 tons broken coal at $6.25 per ton. Make out Bill and Receipt for Mr. Reich, who wishes to pay it at once. *Ans.* $

300. January 12, 1883, Devenney & Son bought of John Wanamaker, Philadelphia, the following goods: 84 pieces cassimere, each piece containing 32 yards, at $2.25 per yard; 19 pieces cheviot, each piece containing 24 yards, at $3.75 per yard; 47 pieces black broadcloth at $75 each piece; and 168

yards Irish linen at $2.75 per yard. Devenney & Son paid $5000 on account. Make out Bill, showing what is still due to John Wanamaker. *Ans.* $

The First Part of our SHORT ARITHMETIC ends here, having completed the study of the First FOUR RULES. Part Second will treat of FRACTIONS, on which, of course, no pupil will enter until he has pretty well mastered Part First. To test his knowledge, the following two sets of Questions have been prepared: one to be answered orally in presence of the whole class; the other to be worked out on the slate within a limited time, and then to be entered neatly in the copy-book.

EXAMINATION QUESTIONS.

FIRST SET.

(To be answered orally.)

1. What is the difference between a Concrete and an Abstract number?

The Pupil should always accompany his answer with an example. Without an appropriate example, words have little or no meaning. This should be remembered in answering all the following questions.

2. The difference between Numeration and Notation?

3. How did the Romans express numbers?

4. Is the Roman method much employed at present?

No; the Roman method is at present employed only to denote chapters in books, figures on clock or watch dials, or generally for the sake of producing a little variety.

5. What is the great principle of the Arabic method?

6. Name ten ranks of figures, beginning with the lowest.

7. Name ten periods of figures in order, beginning with the lowest.

8. Give a short way to numerate.

9. Give a short way to notate.

10. When do we "carry" figures in Addition?

11. When do we "borrow" in Subtraction?

12. When is Multiplication preferable to Addition?

13. What is the use of the Nought?

14. Name and explain the three terms used in Subtraction.

15. Name and explain the three terms used in Multiplication.

16. Explain the difference between "Sum" and "Product."

17. Explain what is meant by the term "Factor."

18. When is Division to be preferred to Subtraction?

19. Explain the three terms employed in Division.

20. In comparing Multiplication with Division, with what term does "Product" correspond?

21. With what do the "Divisor" and the "Quotient" correspond?

22. When the Product and one Factor are given, how is the other Factor found?

23. When the Divisor and the Quotient are given, how is the Dividend found?

24. Give the signs of the Four Rules, explaining each one separately.

25. Give a familiar example showing why 5 times 7 is the same in product as 7 times 5.

26. How is Addition proved? Subtraction? Multiplication? Division?

27. In proving a Division question by Multiplication, what is to be done with the Over?

28. What is the difference between Short and Long Multiplication? Between Short and Long Division?

29. Give an example showing the peculiarity of the figure 9. Can you explain why it has this peculiarity?

30. In how many cases may the Nine Test fail?

Principally in three: first, if a Nine slips in or out accidentally; second, if a Nought slips in or out by accident;

third, when a mistake made in one part of the operation exactly balances a mistake made in another part. But since such accidents are very unlikely to occur, the Nine Test is very valuable, and should be often employed.

EXAMINATION QUESTIONS.

(To be done carefully on the slate and entered neatly in the copy-book within a certain specified time.)

1. Express in the Arabic figures the following Roman numbers: LXXIV, CDLV, MDXL, XCIV, $\overline{\text{X}}$CDXL, $\overline{\text{D}}$CCCLXV, MDCCCLXXXI, $\overline{\text{MXX}}$LVI, $\overline{\text{V}}$DCCLXXIII, $\overline{\text{XVI}}$.

2. Express in Roman numbers the following: 79, 798, 7896, 26, 206, 20068, 68245, 89084, 1884, 367801.

3. Write out the Numeration of the following: 6152178, 717000305, 26000543, 1543286, 168472695, 10000004, 3070901050, 5162738405, 7843000023, 12345678901.

4. Add the following: six hundred and sixty-six thousand and twenty-seven. Four hundred and fourteen thousand and eighty-one. Three hundred and ninety-two thousand, four hundred and three. Five hundred and seventy-three thousand, two hundred and ten. Seven hundred thousand and eighty-seven. Eighty-one millions, three hundred and eighty-four thousand, six hundred and seventy-eight. Seventy-nine millions, six hundred and fifty-four thousand, and one. Sixty-one millions, eight hundred and seventy-two. Fifty-six millions, four hundred and three thousand, eight hundred and six. Thirty-one millions, two hundred and eight thousand, seven hundred and sixty. Prove by the second way as given on page 16.

5. Find the sum of nine billions, eight hundred and seventy-four millions, three hundred and twenty-four thousand, three hundred and sixty-five; and six millions, six thousand, and six. Also find the sum of sixteen billions, five hundred and seventy-five millions, four hundred and twenty-eight thousand,

three hundred and twenty-nine; and one billion, one thousand, and one. Find the difference between the two sums. (Prove.)

6. 87643287 × 34567893 = ? Prove in two ways: first, by the Nines; then by dividing the product by the multiplier.

7. Divide 357764036 by 23678965, and prove, first, by the Nines, and then by Multiplication.

8. Multiply 59872 by 76938. From the product subtract 606431936; divide the remainder by 248, and to the quotient add the difference between 8046875 and 17005.

9. In a mile there are 5280 feet. How many miles are there in 1050720 feet. (Prove.)

10. The Dividend is 685872 and the Quotient is 1732. What is the Divisor? (Prove.)

11. Find the sum of the half, the third, the fourth, the fifth, the sixth, and the seventh of 840.

12. To shoe a horse, 28 nails are required. For how many horses are 70952 nails sufficient? (Prove.)

13. A pound troy contains 5760 grains; a pound avoirdupois contains 7000 grains. In 4032000 grains, how many more pounds troy than pounds avoirdupois? (Prove.)

14. Divide the twelfth part of 1728 apples between 13 boys, and keep what remains over for yourself. What is your share? (Prove.)

15. The circumference of a bicycle wheel is 16 feet. How many times will it turn on the road between two towns 9 miles apart, a mile containing 5280 feet? (Prove.)

16. If 144 men are required for 768 days to cut through a mountain, in how many days would 432 men be able to finish the same task?

17. If 144 men take 768 days to perform a piece of work, how many more men are required so that the work may be finished in 216 days? (Prove.)

18. If 8 men can do a piece of work in 9 days, in how many days could 6 boys do it, considering the work of 2 boys to be equal to that of one man? (Prove.)

19. If 3 men can earn $1200 in 8 months, in what time can 12 men earn the same money? (Prove.)

20. If a car runs 600 miles in 24 hours, how many miles will it run in 11 hours? (Prove.)

21. A book has 360 pages. Each page has 29 lines, and each line has 45 letters. There are 40 chapters in the book, and each begins and ends with half a page. The printer's charges are at the rate of one dollar for every 2880 letters. What does the printing of the book cost? *Ans.* $145.

22. The skins of 6 seals are needed for making a lady's cloak, and 7 sable skins are required to trim it. Estimating 33600 to be the number of sealskin cloaks sold in Philadelphia within the last 25 years, how many animals are sacrificed every year to satisfy the fashionable demand?

23. James Murphy bought of Thomas Edwards, 234 Pine Street, St. Louis, March 12, 1882: 20 pounds coffee at 40 cents a pound; 10 pounds tea at $1.20 a pound; 24 pounds sugar at 12 cents a pound; 36 yards muslin at 22 cents a yard; 27 yards flannel at 50 cents a yard; 15 yards shirting at 18 cents a yard; and 3 dozen handkerchiefs at $2.50 a dozen. Make out Bill.

24. August 13, 1876, Messrs. Bainbridge & Co. bought of Gilmor & Simms, 219 Pearl Street, New York, as follows: 25 barrels flour at 14 dollars per barrel; 125 pounds pearl starch at 15 cents per pound; 16 barrels Chicago beef at $21 per barrel; and 45 boxes cheese at $3.75 per box. Make out Bill and Receipt.

25. Kerr & Co., 1267 Chestnut Street, Philadelphia, sold, April 7, 1884, the following to Mrs. James Wilcox: 2 dozen Bohemian goblets at $7.50 per dozen; 2 soup tureens at $4.50 each; half dozen glass pitchers at $12.80 per dozen; and 6 dozen stoneware plates at $1.56 per dozen. Make out Bill and Receipt.

26. Trenton, January 6, 1874, Mr. D. Odiorne bought of S. P. McFillin, 45 Main Street: 18 yards drilling at 16 cents a yard; 3 dozen handkerchiefs at 50 cents a dozen; 25 yards

flannel at 75 cents a yard; and 84 yards sheeting at 32 cents a yard. Make out Bill.

27. Philadelphia, April 8, 1882, G. W. Childs bought of Claxton & Co. the following books: Grove's Dictionary of Music, 4 volumes, at $5.75 per volume; Rawlinson's Monarchies, 9 volumes, at $3.75 per volume; Smith's Dictionaries, 15 volumes, at $9.75 per volume; and Guizot's History of France, 5 volumes, at $6.25 per volume. Make out Bill.

28. Albany, December 20, 1881, Edward Nolan bought of Simon Martin, 137 Brown Street, as follows: 35 pounds raisins at 18 cents per pound; 25 pounds turkey at 15 cents a pound; 15 pounds minced meat at 20 cents a pound; and 3 barrels Genesee flour at $7.65 a barrel. Bill and Receipt.

29. Newport, R. I., July 9, 1883, Henry White bought of J. B. Sword, 27 Thames Street: 12 drums figs at $3.25 per drum; 16 barrels apples at $6.25 per barrel; 50 pounds fresh mackerel at 12 cents a pound; and 2 firkins butter, weighing together 112 pounds, at 30 cents a pound. On the same day Sword bought of White: 40 yards calico at 9 cents a yard; 150 yards sheeting at 17 cents a yard; 146 yards Irish linen at 65 cents a yard; and 12 pieces velvet ribbon at $1.75 per piece. Make out both Bills, and tell to whom the cash balance is due.

30. Philadelphia, June 1, 1883, James Dornan, butcher, sent the following statement to William Templeton, grocer: 62 pounds prime beef at 20 cents a pound; 75 pounds tenderloin at 25 cents a pound; 64 pounds lamb at 18 cents a pound; 57 pounds veal at 16 cents a pound; 24 sweetbreads at 25 cents each; 8 dozen kidneys at $1.20 per dozen. On the same day Mr. Templeton sent Mr. Dornan a statement with the following items: 84 pounds sugar at 11 cents a pound; 150 pounds butter at 42 cents per pound; 75 pounds tea at 90 cents per pound; 80 pounds coffee at 37 cents a pound; 120 pounds cornmeal at 2 cents a pound; and 240 pounds buckwheat at 3 cents a pound. Make out both Bills, and tell how much cash is to be paid to settle the account, and who is to pay it.

END OF THE FOUR RULES.

APPENDIX.

EXTRA SUMS

For pupils who are a little too quick for the regular class.

If these are entered in the copybook, they should be called EXTRA QUESTIONS, and counted by their own numbers. Or they may be called EXTRA SUMS, the word *sum* being a handy term for a question in arithmetic to be worked out on slate or paper.

QUESTION TABLE.

	1	2	3	4	5	6	7	8	9	10	11	12
1	3	9	8	7	6	7	4	2	9	0	3	7
2	3	5	4	2	9	0	7	4	9	6	3	0
3	6	0	3	8	4	6	1	7	8	8	2	8
4	2	9	1	7	5	4	3	9	1	7	7	4
5	8	9	3	7	0	2	0	3	1	2	3	5
6	5	0	1	3	6	7	4	4	5	6	3	2
7	5	1	7	5	2	3	0	6	4	3	2	5
8	3	4	6	1	4	3	2	1	9	7	9	0
9	3	7	8	3	7	0	6	8	8	1	4	4
10	7	2	9	5	8	1	8	2	7	2	0	9
11	6	0	5	9	4	3	0	3	4	7	6	3
12	2	5	9	0	4	9	8	2	2	9	7	5

The vertical COLUMNS, or those running up and down, are numbered from 1 to 12. The horizontal LINES, or those running right and left, are also numbered from 1 to 12.

When a Column is treated as a Line, the undermost figure is the Unit.

EXTRA SUMS IN ADDITION.

1. Add lines 1 to 6, and prove.

1 to 6 means the *first*, *second*, *third*, *fourth*, *fifth*, and *sixth*. In such expressions the *first* and the *last* are always included.

2. Add lines 3 to 8, and prove (as on page 16).
3. Add lines 5 to 10, and prove (as on page 16).
4. Add lines 7 to 12, and prove (as on page 16).
5. Add lines 4 to 9, and prove (as on page 16).
6. Add lines 6 to 11, and prove (as on page 16).
7. Add lines 2 to 8, and prove (as on page 16).
8. Add lines 4 to 12, and prove (as on page 16).
9. Add all the odd lines, and prove (as on page 16).
10. Add all the even lines, and prove (as on page 16).
11. Add columns 1 to 6, and prove (as on page 16).
12. Add columns 3 to 8, and prove (as on page 16).
13. Add columns 5 to 10, and prove (as on page 16).
14. Add columns 7 to 12, and prove (as on page 16).
15. Add columns 4 to 9, and prove (as on page 16).
16. Add columns 6 to 11, and prove (as on page 16).
17. Add columns 2 to 8, and prove (as on page 16).
18. Add columns 4 to 11, and prove (as on page 16).
19. Add odd columns, and prove (as on page 16).
20. Add even columns, and prove (as on page 16).
21. Add lines 1 to 4 to columns 1 to 4, and prove.
22. Add lines 2 to 6 to columns 2 to 6, and prove.
23. Add lines 3 to 7 to columns 3 to 7, and prove.
24. Add lines 4 to 8 to columns 4 to 8, and prove.
25. Add lines 5 to 9 to columns 5 to 9, and prove.

Remember, in treating Columns as Lines, the *undermost* figure is the Unit, from which the others take their value.

EXTRA SUMS IN SUBTRACTION.

26. Find difference of lines 1 and 2, and prove.
27. Find difference of lines 3 and 4, and prove.
28. Find difference of lines 5 and 6, and prove.
29. Find difference of lines 7 and 8, and prove.
30. Find difference of lines 9 and 10, and prove.
31. Find difference of lines 11 and 12, and prove.
32. Find difference of lines 1 and 3, and prove.
33. Find difference of lines 2 and 5, and prove.
34. Find difference of lines 3 and 6, and prove.
35. Find difference of lines 4 and 7, and prove.
36. Find difference of column 1 and line 2; prove.
37. The difference between the sum of columns 3 and 4 and that of lines 3 and 4, and prove.
38. Difference between sum of columns 5 and 6 and that of lines 5 and 6; prove.
39. Difference between sum of columns 7 and 8 and that of lines 7 and 8; prove.
40. Difference between sum of columns 9 and 10 and that of lines 9 and 10; prove.
41. Difference between sum of columns 11 and 12 and that of lines 11 and 12; prove.
42. Difference between sum of columns 1, 3, and 5 and that of lines 1, 3, and 5; prove.
43. Difference between sum of columns 2, 4, and 6 and that of lines 2, 4, and 6; prove.
44. Difference between sum of columns 3, 5, and 7 and that of lines 3, 5, and 7; prove.
45. Difference between sum of columns 4, 6, and 8 and that of lines 4, 6, and 8; prove.
46. Difference between sum of columns 5, 7, and 9 and that of lines 5, 7, and 9; prove.
47. Difference between sum of columns 6, 8, and 10 and that of lines 6, 8, and 10; prove.

48. Difference between sum of columns 7, 9, and 11 and that of lines 7, 9, and 11; prove.

49. Difference between sum of columns 8, 10, and 12 and that of lines 8, 10, and 12; prove.

50. Difference between sum of columns 7, 9, 10, 12 and that of lines 7, 9, 10, 12; prove.

EXTRA SUMS IN MULTIPLICATION.

51. Multiply line 1 by 5, and prove by the nines.
52. Multiply line 1 by 12, and prove by the nines.
53. Multiply line 1 by 19, and prove by the nines.
54. Multiply line 1 by 36, and prove by the nines.
55. Multiply line 1 by 52, and prove by the nines.
56. Multiply line 2 by 79, and prove by the nines.
57. Multiply line 2 by 84, and prove by the nines.
58. Multiply line 2 by 137, and prove by the nines.
59. Multiply line 2 by 268, and prove by the nines.
60. Multiply line 2 by 796, and prove by the nines.
61. Multiply line 3 by 15, and prove by division.
62. Multiply line 3 by 49, and prove by division.
63. Multiply line 3 by 87, and prove by division.
64. Multiply line 3 by 165, and prove by division.
65. Multiply line 3 by 374, and prove by division.
66. Multiply line 3 by 482, and prove by division.
67. Multiply line 4 by 796, and prove by division.
68. Multiply line 4 by 1865, and prove by division.
69. Multiply line 4 by 2745, and prove by division.
70. Multiply line 5 by 3676, and prove by division.
71. Multiply line 5 by 4758, and prove by division.
72. Multiply line 5 by 7064, and prove by division.
73. Multiply line 6 by 8902, and prove by division.
74. Multiply line 6 by 9576, and prove by division.
75. Multiply line 7 by 91728, and prove by division.

EXTRA SUMS IN DIVISION.

76. Divide line 8 by 5, proving by the nines.
77. Divide line 8 by 12, proving by the nines.
78. Divide line 8 by 17, proving by the nines.
79. Divide line 8 by 48, proving by the nines.
80. Divide line 8 by 364, proving by the nines.
81. Divide line 6 by 732, proving by multiplication.
82. Divide line 7 by 894, proving by multiplication.
83. Divide line 8 by 971, proving by multiplication.
84. Divide line 9 by 1096, proving by multiplication.
85. Divide line 10 by 1348, proving by multiplication.
86. Divide line 11 by 1728, proving by multiplication.
87. Divide line 12 by 3456, proving by multiplication.
88. Divide column 1 by 4029, proving by multiplication.
89. Divide column 2 by 5365, proving by multiplication.
90. Divide column 3 by 7965, proving by multiplication.
91. Divide column 4 by 8378, proving by multiplication.
92. Divide column 5 by 9482, proving by multiplication.
93. Divide column 6 by 12372, proving by multiplication.
94. Divide column 7 by 15643, proving by multiplication.
95. Divide column 8 by 21376, proving by multiplication.
96. Divide column 9 by 25294, proving by multiplication.
97. Divide column 10 by 89453, proving by multiplication.
98. Divide column 11 by 123964, proving by multiplication.
99. Divide column 12 by 298567, proving by multiplication.
100. Multiply line 6 by line 7, and prove both by the nines and multiplication.

In working out this long sum, the pupil should test every step by the nines. Even a slight mistake would give a vast amount of unnecessary trouble.

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1	2	3	4	5	6	7	8	[illegible]	10	11	12	13	14	15	16	17	18	19	20	21	22	[illegible]	24
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120	126	132	138	144
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140	147	154	161	168
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160	168	176	184	192
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180	189	198	207	216
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220	231	242	253	264
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260	273	286	299	312
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280	294	308	322	336
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340	357	374	391	408
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360	378	396	414	432
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380	399	418	437	456
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480
21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420	441	462	483	504
22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440	462	484	506	528